Turbulence and dispersion in atmospheric flows

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Overview

1. Basics on turbulence
2. Observations in atmosphere
3. Dispersion problems in turbulence
4. Lagrangian stochastic modelling

Turbulence basics

Navier-Stokes equations

Turbulence is completely contained in the Navier-Stokes equations (conservation of momentum) for incompressible fluid

$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -\frac{1}{\rho}\frac{\partial p}{\partial x_i} + \nu \frac{\partial^2 u_i}{\partial x_j \partial x_j}$$

$$\frac{\partial u_i}{\partial x_i} = 0$$

If U is the scale of velocity variation and L is the scale over which this variation occurs, the relative weight of the non-linear term $u_j \partial_j u_i$ and the viscous term $\nu \partial_i \partial_i u$ is measured by the Reynolds number

$$\text{Re} = \frac{UL}{\nu}$$

Large Reynolds number $Re \gg 1$ causes non-linearities to make the non-linear term dominating over the viscous one and this makes the flow unpredictable.

Typical values of Re for atmospheric/oceanic flows are larger than 10^8.

Turbulence: many scales

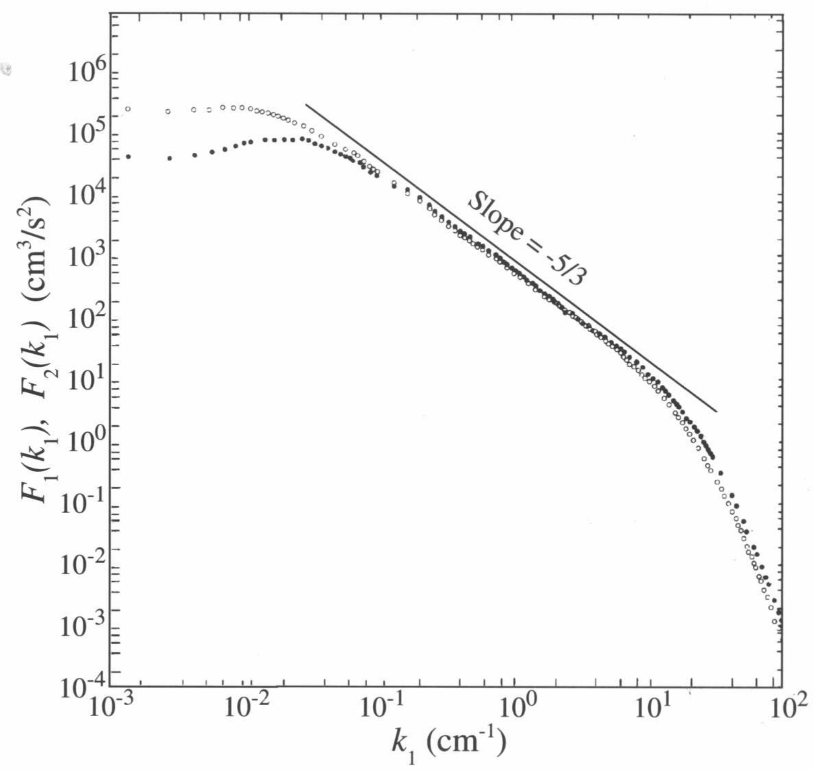


Fig. 5.7. log–log plot of the energy spectra of the streamwise component (white circles) and lateral component (black circles) of the velocity fluctuations in the time domain in a jet with $R_\lambda = 626$ (Champagne 1978).

(after Frisch, 1995)

Energy is injected at large scales and dissipated by molecular viscosity at small scales

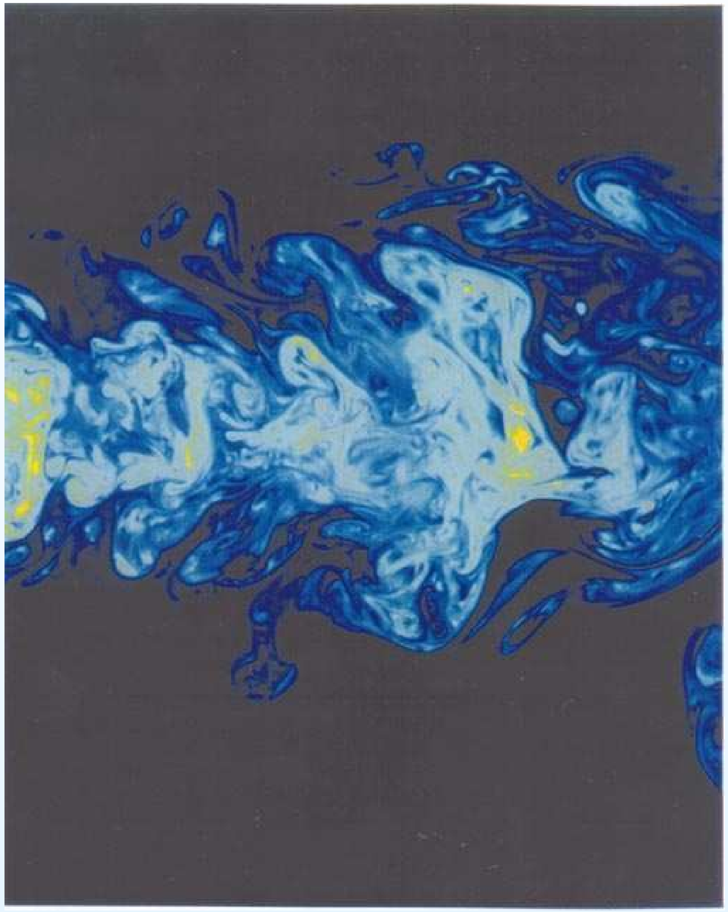
Kolmogorov (1941), based on simple scaling arguments and on the invariance of the N-S equations, fround that for scales ℓ, $L \gg \ell \gg \eta = (\nu^3/\varepsilon)^{1/4}$, the Energy spectrum of turbulence is

$$E(k) \propto \varepsilon^{2/3} k^{-5/3}$$

The number of active scales in this range is of the order of $Re^{9/4}$! (see, e.g., Tennekes and Lumley, 1972)

Navier-Stokes equations cannot be solved in general (not even in particular!).

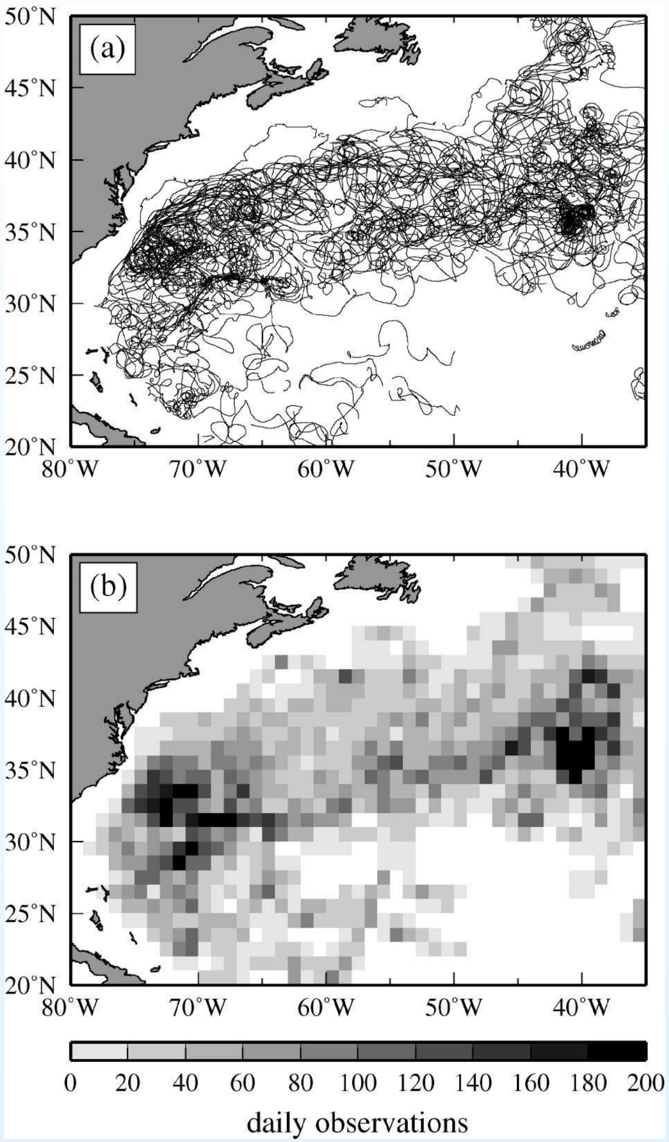
Turbulence: mixing properties



Fluorescent dye in a turbulent jet (Shraiman and Siggia, 2000).

Mixing is enhanced by non-linear instabilities. If compared to molecular mixing, turbulent mixing is Re orders of magnitude larger.

Geophysical mixing examples: ocean



“Spaghetti plot” of drifters in the Atlantic Ocean (Veneziani et al., 2004)

Geophysical mixing examples: free troposphere

Etna 2002 eruption:
MODIS image on 2002-11-02.

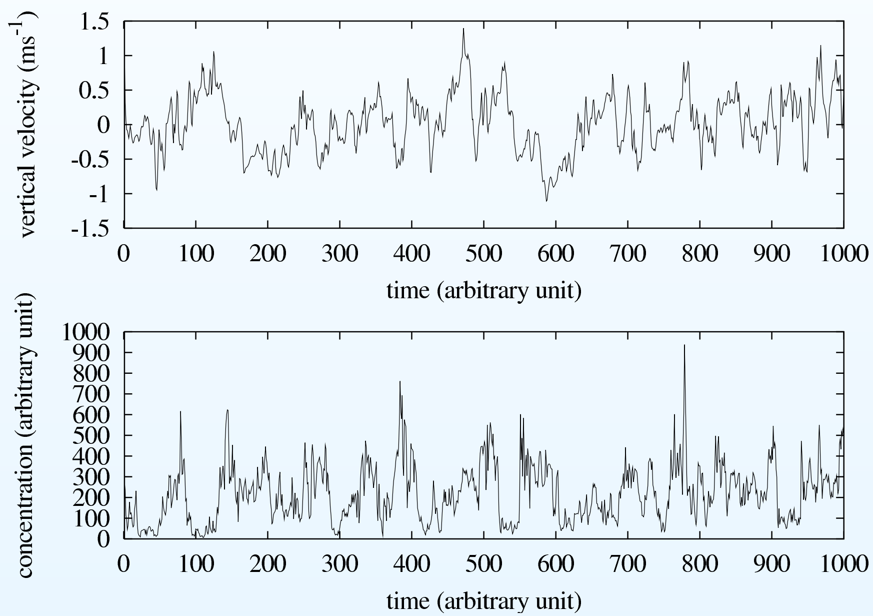
See also the animation of the thermal view of the Etna plume from 2002-10-27 to 2002-11-04 (courtesy of Fred Prata).

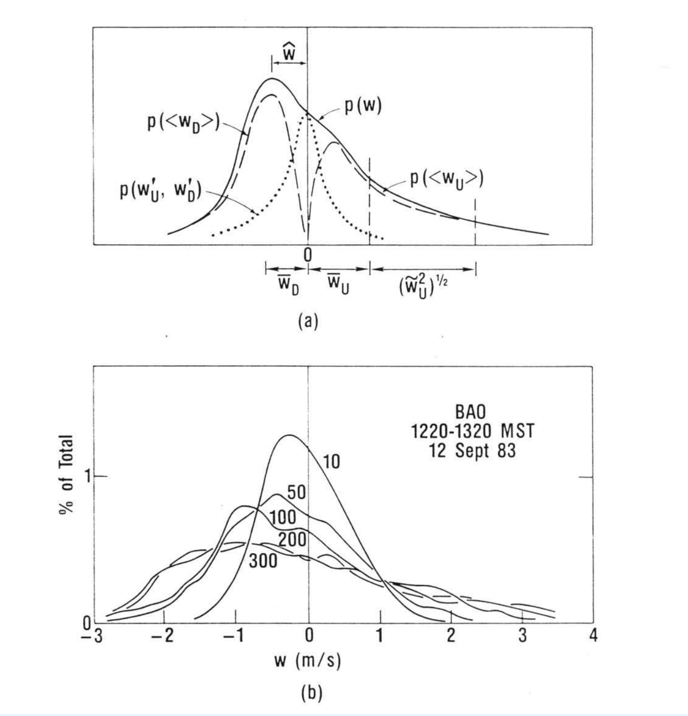
Another mixing examples . . .

This is to show that mixing accurs at very different scales, but with same properties.

Turbulence as a stochastic process

Reynolds (1883) proposed to describe a turbulent flow in terms of average quantities.





Turbulent quantities are seen as stochastic variables and then one can describe them in terms of statistical properties, e.g., it is possible to think of a probability density function (PDF) of the variable.

Reynolds average

Reynolds average: $\langle\cdot\rangle = \frac{1}{N}\sum(\cdot)$

Reynolds averaging rules:

- Given a turbulent quantity a, this is decomposed in a mean part $A = \langle a\rangle$ and a fluctuating part $a' = a - A$
- by definition $\langle a'\rangle = 0$
- any term of the type $a'A$ has null mean value

Remember that Reynolds average is an “ensemble” average but in practical applications time average is performed instead.

$$\langle\cdot\rangle_T = \frac{1}{T}\int_0^T (\cdot)\,\mathrm{d}t$$

This implies the assumption of ergodicity of the process: be careful!

Reynolds equations

Applying Reynolds average rules to the Navier-Stokes equations one obtains

$$\frac{\partial \langle u_i \rangle}{\partial t} + \langle u_j \rangle \frac{\partial \langle u_i \rangle}{\partial x_j} = -\frac{1}{\rho_{00}} \frac{\partial \langle p \rangle}{\partial x_i} + \nu \frac{\partial^2 \langle u_i \rangle}{\partial x_j \partial x_j} - \frac{\partial \langle u_i' u_j' \rangle}{\partial x_j}$$

and for continuity

$$\frac{\partial \langle u_i \rangle}{\partial x_i} = 0$$

$$\frac{\partial u_i'}{\partial x_i} = 0$$

Equations above are “exact” but not closed: first-order moments depend on second order moments as a consequence of the non-linear term in the N-S equations.

This is more general: any-order Re-averaged equation will depend on any-order+1 moments. This gives rise to the closure problem

This is true also for PDF: from N-S equations it is possible to derive an equation for the one-point PDF. This will depend on the two-point PDF!

Clusure problem and closure models

Following an idea of Boussinesq (1877) further developped by G. Taylor and L. Prandtl, the closure problem can be “solved” using an analogy with the molecular viscosity: microscopic scale motion has the macroscopic consequence of producing a viscosity.

In analogy, small-scale eddies could be thought to act on large-scale eddies in a diffusive manner.

This leads to the concept of “eddy viscosity”: $\nu_{\text{eddy}} \sim u\ell$ where u is the r.m.s of the turbulent velocity and ℓ is, in analogy with the mean free path, its spatial correlation scale.

It results that

$$\frac{\nu_{\text{eddy}}}{\nu_{\text{mol}}} = \frac{u\ell}{\nu_{\text{mol}}} \sim Re$$

as anticipated.

Thus, correlation (flux) terms can be written in analogy with the Fick law for diffusion:

$$\langle u_i' u_j' \rangle = -\nu_{\text{eddy}} \left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right)$$

Second-order moments Reynolds equations

$$\frac{\partial \overline{u_i' u_k'}}{\partial t} + \overline{u_j} \frac{\partial \overline{u_i' u_k'}}{\partial x_j} = (-\overline{u_i' u_j'} \frac{\partial \overline{u_k}}{\partial x_j} - \overline{u_k' u_j'} \frac{\partial u_i}{\partial x_j}) - \frac{\partial \overline{u_i' u_j' u_k'}}{\partial x_j}$$

$$+ \frac{g}{\vartheta_{00}} (\delta_{k3} \overline{u_i' \vartheta'} + \delta_{i3} \overline{u_k' \vartheta'}) + f(\varepsilon_{kj3} \overline{u_i' u_j'} + \varepsilon_{ij3} \overline{u_k' u_j'}) - \frac{1}{\rho_{00}} \left(\overline{u_k' \frac{\partial p'}{\partial x_i}} + \overline{u_i' \frac{\partial p'}{\partial x_k}} \right)$$

$$+ \nu \frac{\partial^2 \overline{u_i' u_k'}}{\partial x_j \partial x_j} - 2\nu \overline{\frac{\partial u_i'}{\partial x_j} \frac{\partial u_k'}{\partial x_j}}$$

Note that

$$2\nu \langle \frac{\partial u_i'}{\partial x_j} \frac{\partial u_k'}{\partial x_j} \rangle \equiv \varepsilon_{ik} \simeq \frac{2}{3} \delta_{ik} \varepsilon \, , \quad \varepsilon = \nu \langle \left(\frac{\partial u_i'}{\partial x_j} \right)^2 \rangle$$

ε is a (positive) dissipation term for the diagonal terms of the tensor $\langle u_i' u_k' \rangle$.

Summary on turbulence

- turbulence is a non-linear unpredictable phenomenon
- turbulent motion has a huge number of degrees of freedom and a universal behaviour (K41)
- turbulence has strong diffusive properties
- statistical approach lead to the closure problem
- “eddy viscosity”, associated to Fick law (flux-gradient transport) is a useful concept to close the problem

Some observations of atmospheric turbulence

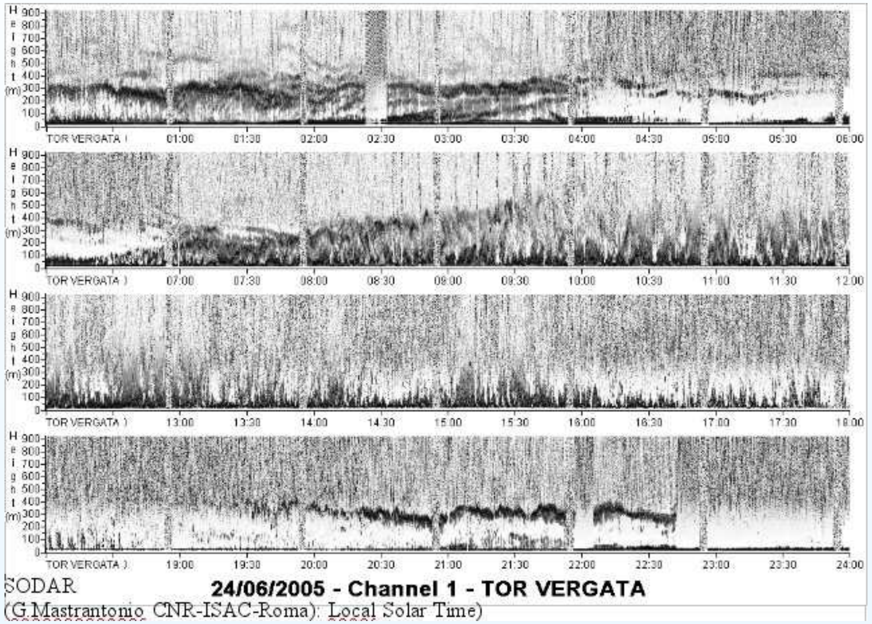
Atmospheric structure

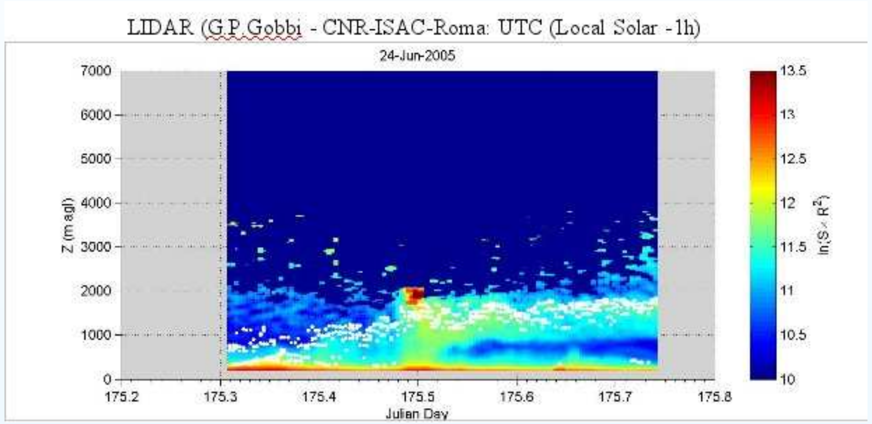
Regions of the atmosphere considered here are

the atmospheric boundary layer (ABL) where turbulence is fully developped (though not isotropic) almost all the time, with variations in static stability. Broadly: unstable during the daytime, neutral/stable during nighttime

the free troposphere (FT) the region above the ABL upt to the tropopause (~ 10 km) where the flow is stably stratified except in localised moint convection cells. Turbulence is highly anisotropic with vertical motions much smaller that horizontal ones ($U \sim 10$ m s^{-1} and $W \sim 1$ cm s^{-1})

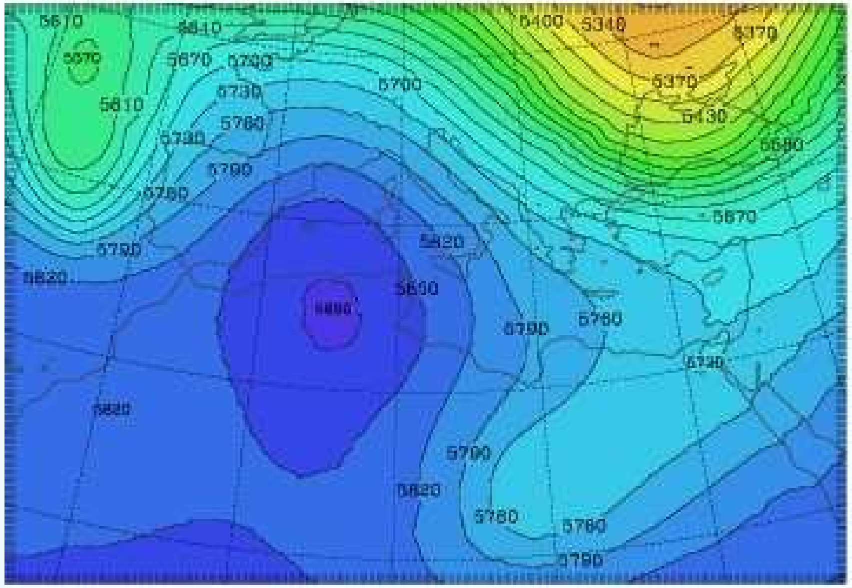
Diurnal cycle in the ABL



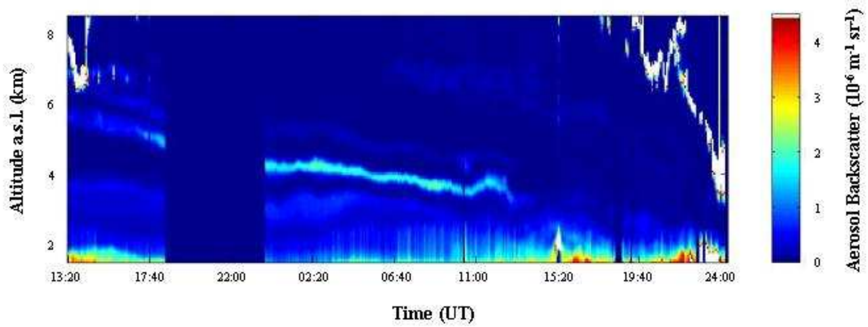


Evolution of the ABL on 2005-06-24 at Tor Vergata (RM), Italy as seen by SODAR (courtesy of G.G.Mastrantonio) and LIDAR (courtesy of G.P.Gobbi).

FT structure

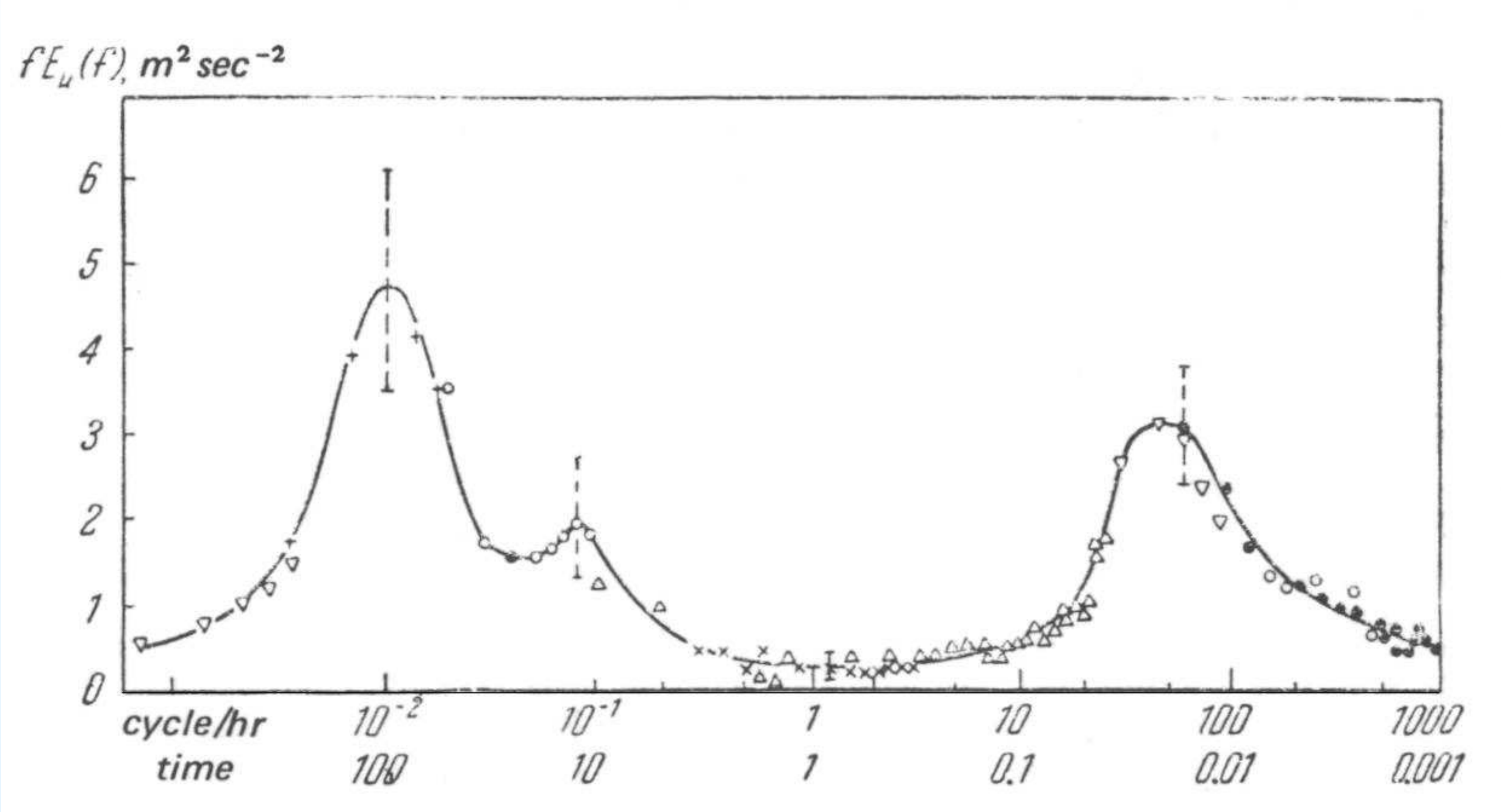


geopotential height at 500 hPa on 2002-10-29 (BOLAM).



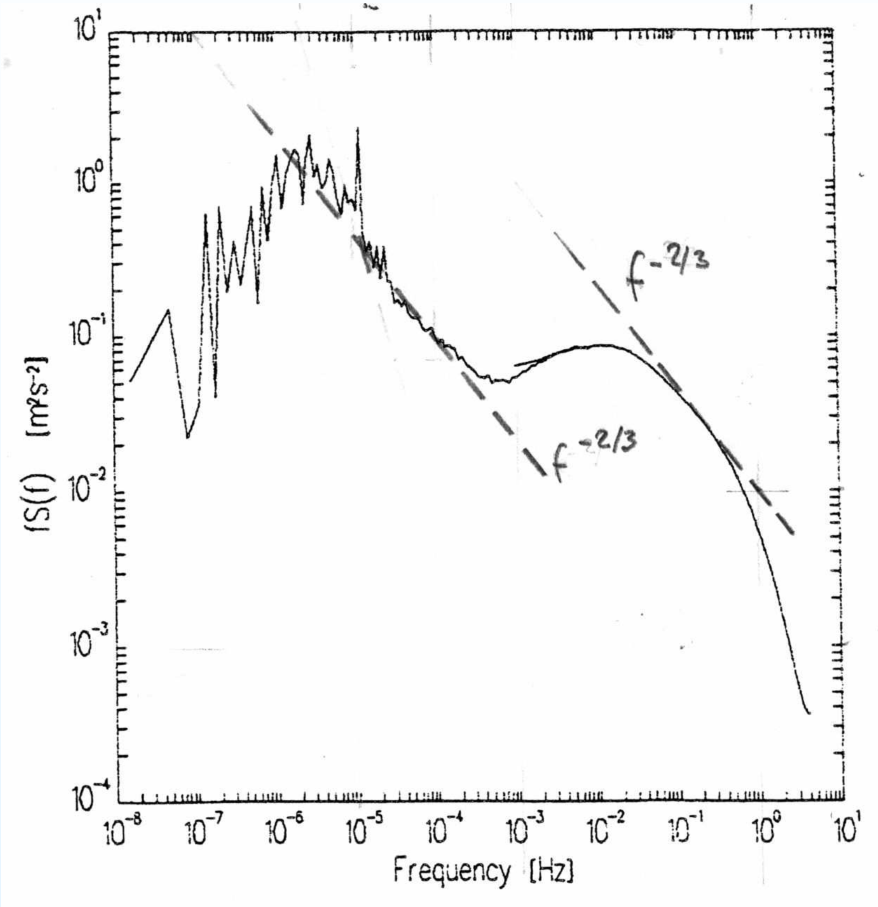
Lidar backscatter map measured at Potenza (Italy) on 2002-11-01 and 2002-11-02.

Atmospheric Spectra: the Atmospheric Bondary Layer



Horizontal velocity variance spectrum in the PBL, from Van der Hoven (1957). Observe the gap between mesoscale and fully 3D boundary-layer turbulence

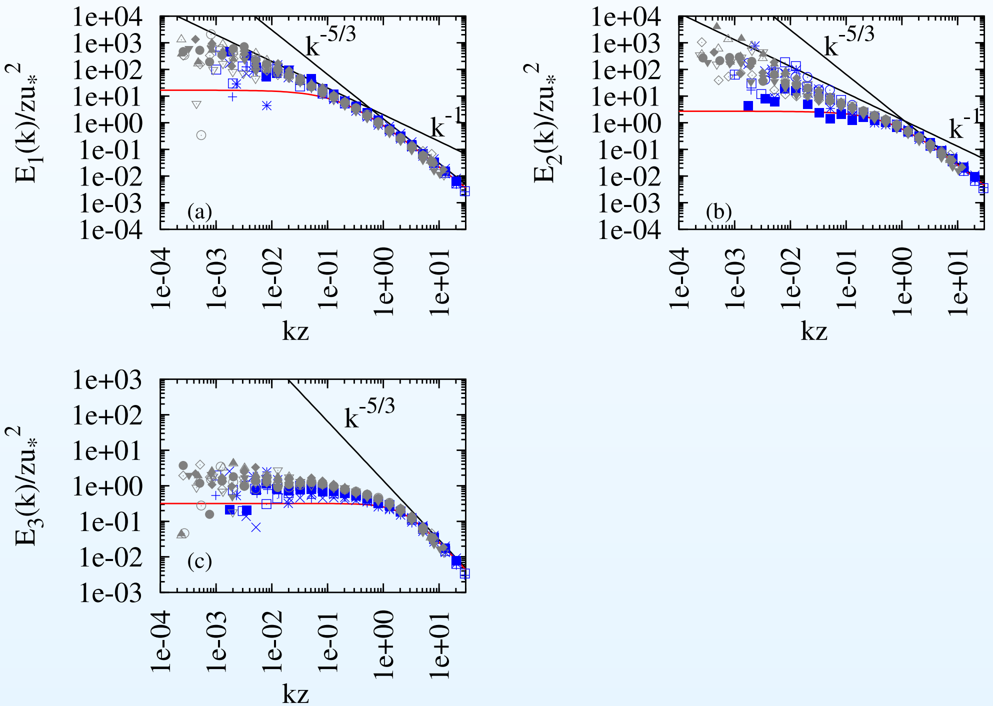
Atmospheric Spectra: different regions of K41



Horizontal velocity variance spectrum in the PBL, from Courtney and Troen (1990).
Observe the presence of two ranges of Kolmogorov turbulence.

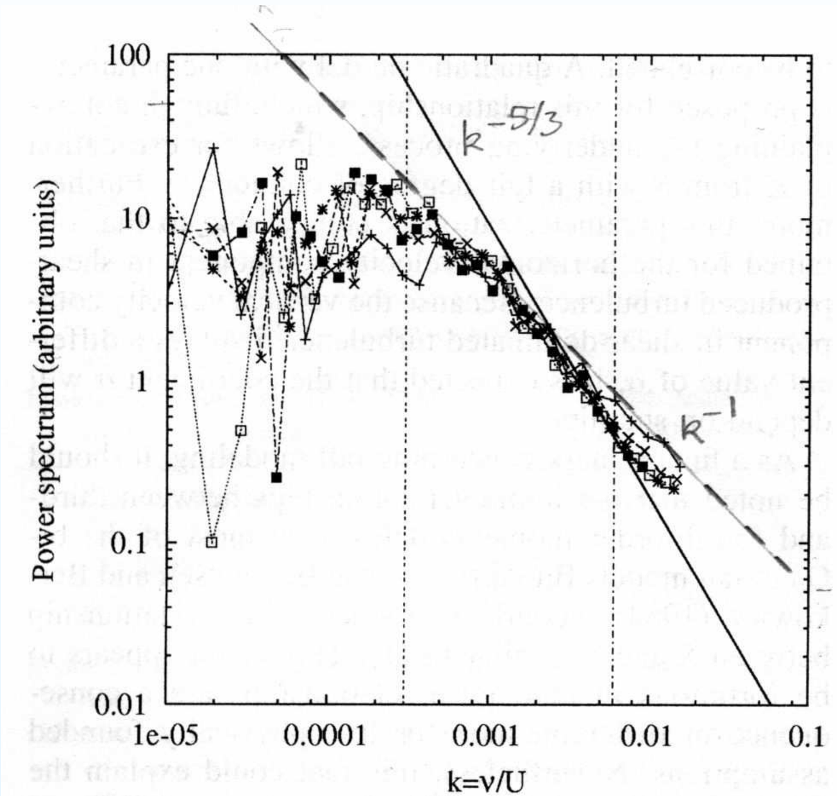
Small scale spectra: Neutral BL

Most of the PBL studies concentrate on the high frequency part of the spectrum:



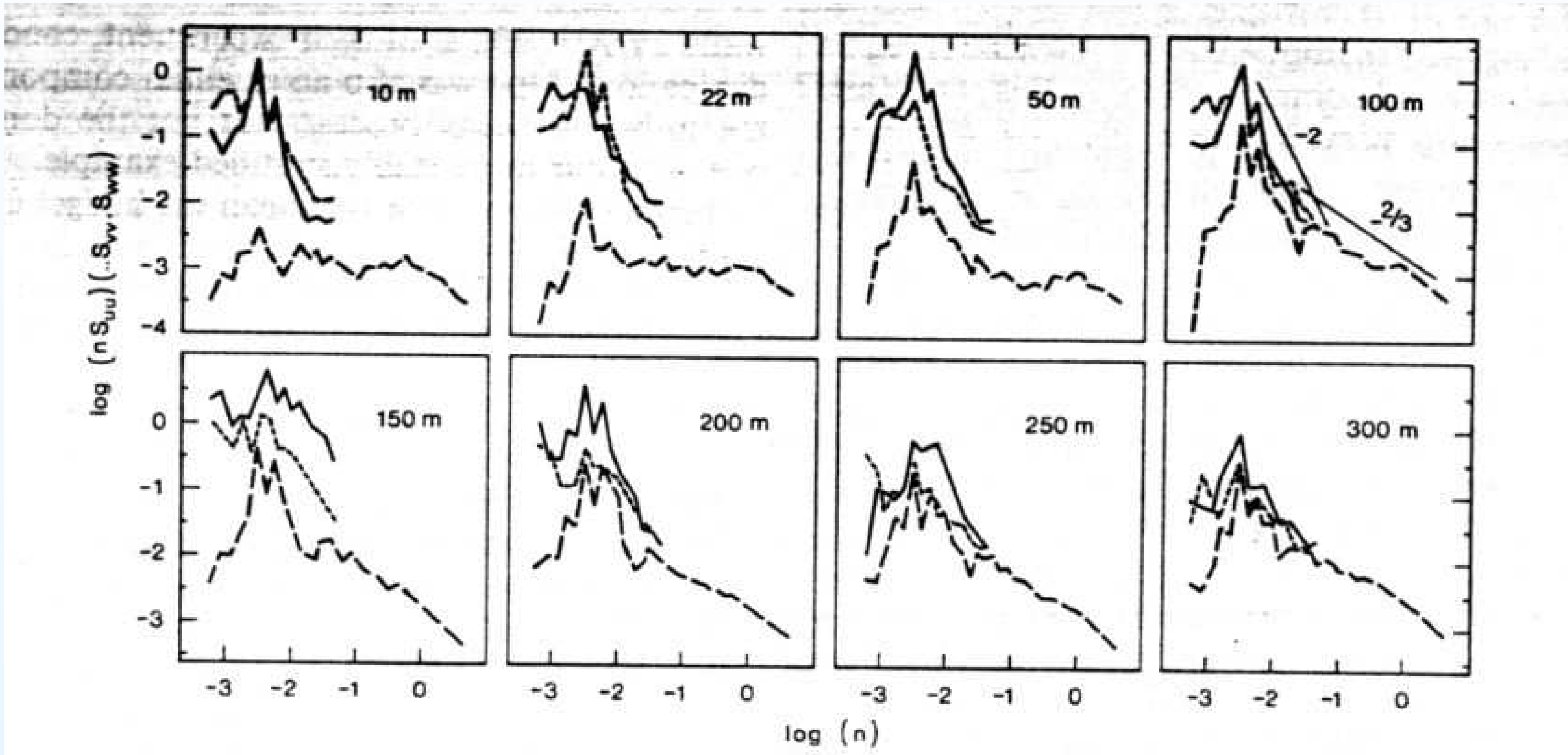
Spectra (single cases reported with different symbols) measured at NIS at 2 m and at 10 m for the (a) longitudinal , (b) lateral and (c) vertical components. The lines show the reference spectra (Kaimal et al., 1972).

Small scale spectra: Convective Boundary Layer



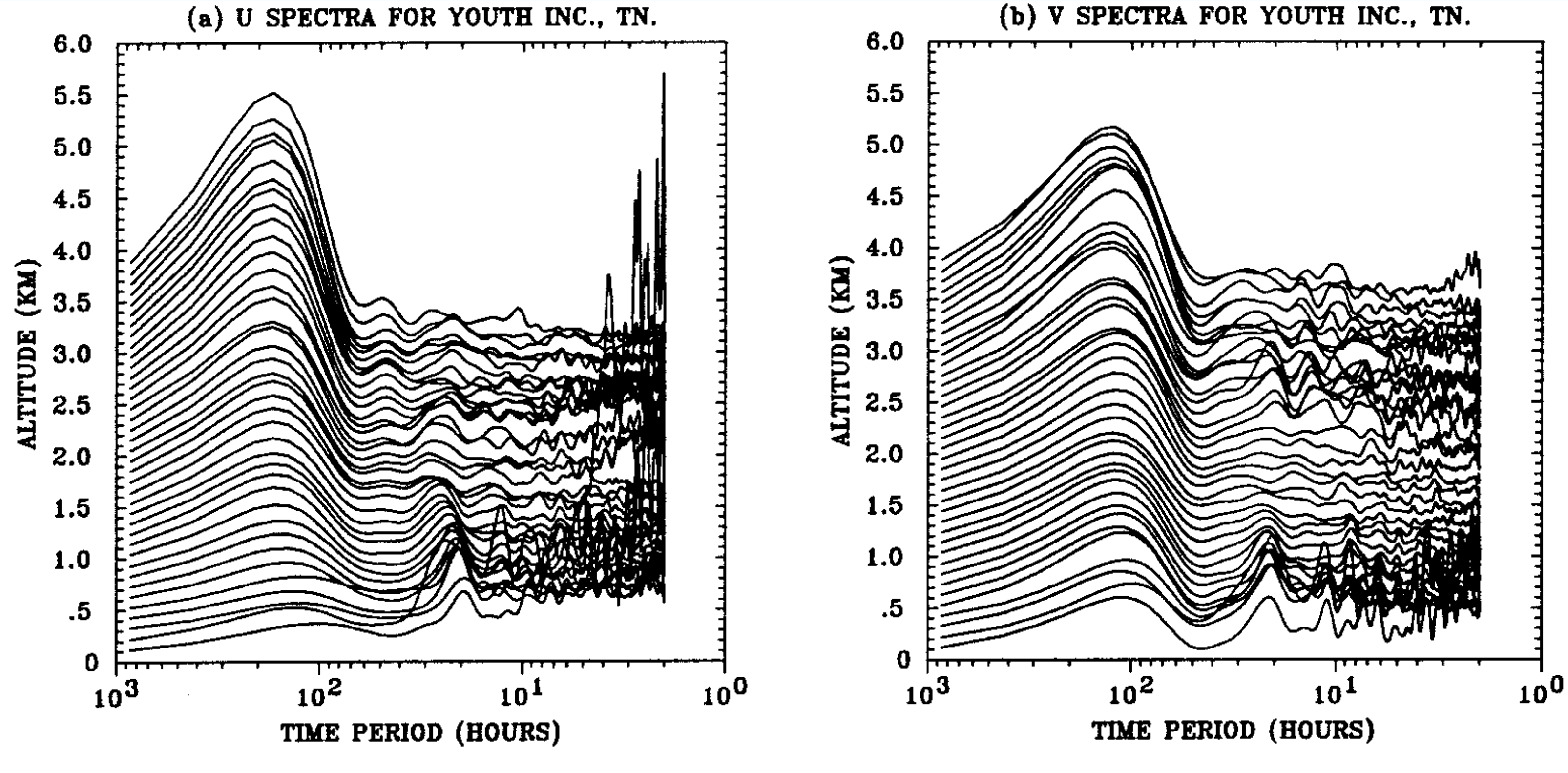
Vertical velocity in the CBL, from Alberghi et al. (2002) show a similar scaling k^{-1} as the horizontal components in the neutral BL.

Small scale spectra: Stable Boundary Layer



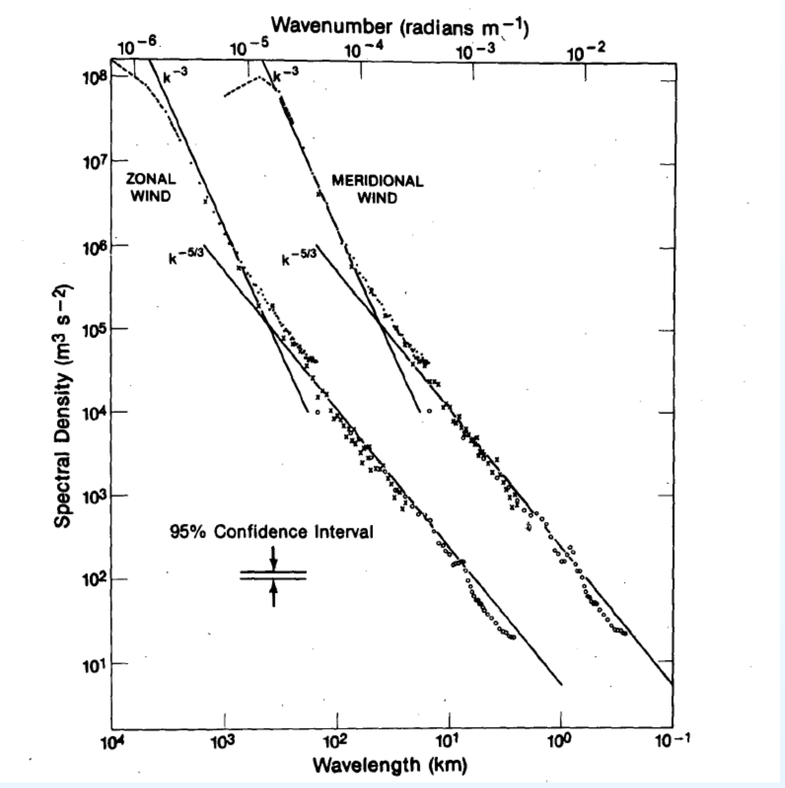
Stably stratified BL: a classical paper by Finnigan et al. (1984) shows that the horizontal components contain more energy than the vertical one in the lower layers, whereas at higher layers turbulence tends to become isotropic, and also shows the slope typical for gravity waves.

Atmospheric spectra: From FT to BL (*and vice versa*)



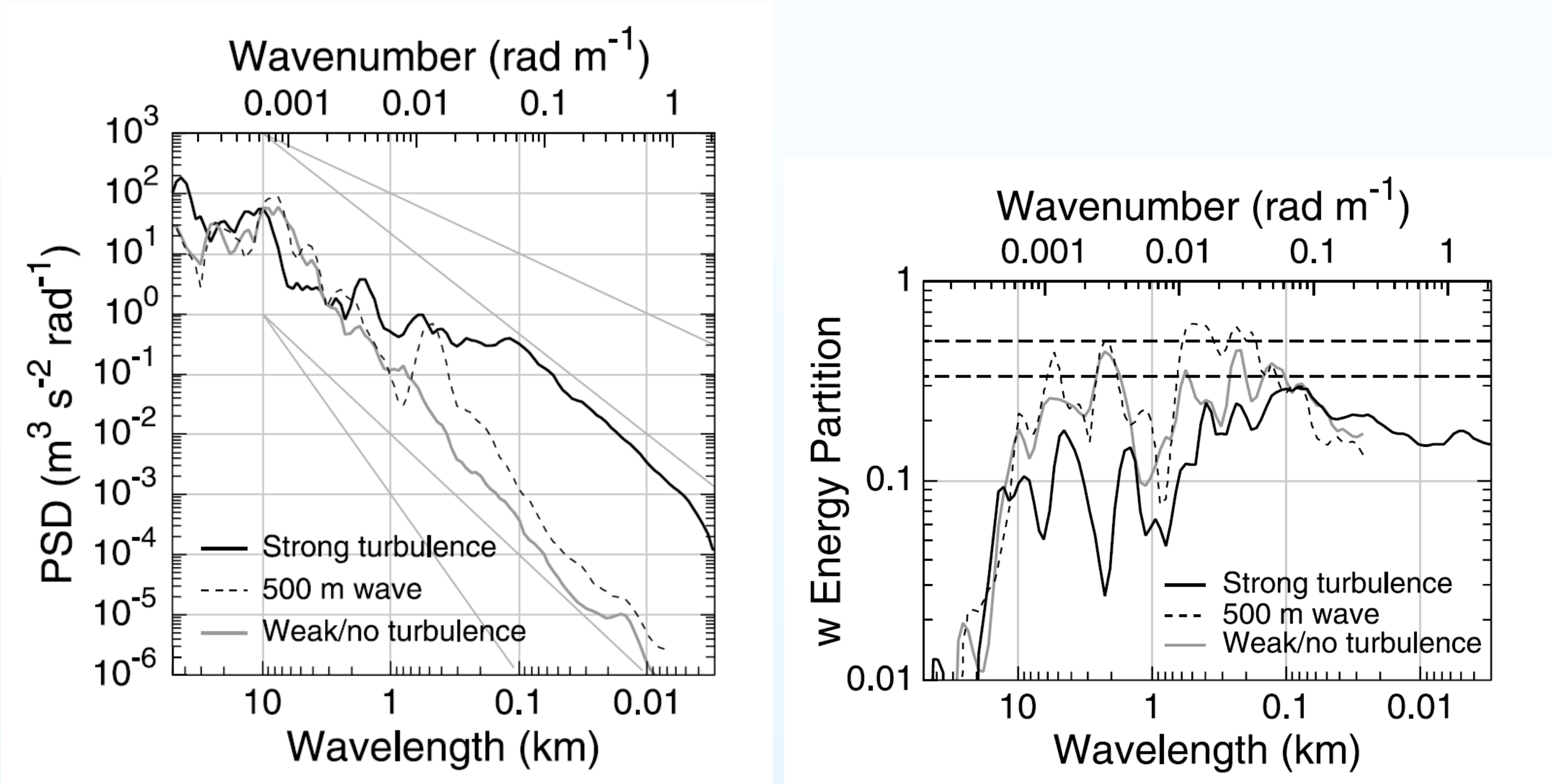
The relative energy spectra of horizontal wind components (u and v) for Dickson. Constants are added to the energy spectra amplitude at successive heights to separate them on the graph. The corresponding height of a spectrum can be read from the left axis. (c) and (d) Horizontal wind component spectra at the lowest (120 m) and 2.05-km range gates for Dickson. (Gupta et al., 1997)

Atmospheric spectra: the Troposphere



Composite of wavenumber spectra of zonal and meridional velocity (shifted one decade on the right, after Nastrom et al. (1984)).

Atmospheric spectra: near the Tropopause



Ensemble-averaged vertical wind speed spectra meassured in a frame of reference relative to air (left) and ratio between vertical component and total wind spectra, showing the departure from isotropy at different wavelenghts, after Duck and Whiteway (2005).

Summary for atmospheric observations

Different mechanisms:

- the balance between momentum and heat flux, and the wall effect, in the surface layer
- shear, convection and gravity waves in the well developed PBL
- deep convection, waves and 2D large scale motions in the troposphere

and all lead to spectra characterized by an inertial-like range.

Dispersion: observations and modelling problems

Dispersion by continuous movements

After Taylor (1921)

(1) $$x_i(t) = \int_0^t v_i(t_1)\,\mathrm{d}t_1 + x_i(0)$$

(2) $$\overline{x_i x_j} = \int_0^t \mathrm{d}t_1 \int_0^t \mathrm{d}t_2 \overline{v_i(t_1) v_j(t_2)} = 2\int_0^t R_{ij}(s)(t-s)\,\mathrm{d}s$$

(3) $$T_{ij} = \frac{1}{\overline{v_i' v_j'}} \int_0^\infty R_{ij}(s)\,\mathrm{d}s$$

Dispersion by continuous movements

For $t << T_{ij}$ the ballistic regime occurs:

(4) $$\overline{x_i x_j} = \overline{v_i(0) v_j(0)} t^2$$

For $t >> T_{ij}$ diffusion occurs:

(5) $$\overline{x_i x_j} \simeq 2 T_{ij} \overline{u'_i u'_j} t$$

and the eddy diffusion coefficient (computed as a limit) is

(6) $$D_{ij} = \lim_{t \to \infty} \frac{\overline{x_i x_j}}{2t} = \overline{u'_i u'_j} T_{ij}$$

Relative dispersion

In the inertial subrange where energy scales as $k^{-5/3}$, the mean square separation scales as

$$\langle \Delta x_i^2 \rangle \propto \varepsilon t^3$$

This is equivalent to have a “diffusion coefficient” that scales as

$$D \propto u\ell \propto \ell^{1/3}\ell = \ell^{4/3}$$

Turbulent diffusion and eddy diffusion coefficient (1)

Starting from the “exact” diffusion equation for the scalar concentration c

$$\partial_t c + \underline{u} \cdot \nabla c = \kappa \nabla^2 c \,,$$

where $\underline{u}$ is the turbulent field (solution of the Navier-Stokes equations), and taking the Reynolds (ensemble) average

$$C = \langle c \rangle \,, \; c = C + c' \,, \, u_i = U_i + u'_i$$

one obtains:

$$\partial_t C + \underline{U} \cdot \nabla C = \nabla \cdot \langle \underline{u}' c' \rangle$$

which is an exact but not closed expression for the (ensemble) mean concentration C

Turbulent diffusion and eddy diffusion coefficient (2)

Assuming the validity of Fick's law in analogy with molecular diffusivity (Boussinesq,1877)

$$\langle u_i' c' \rangle = -D_{ij} \partial_{x_j} C$$

where $D_{ij} = \langle u_i' u_j' \rangle T_L$ is the "eddy" diffusion coefficient, the Reynolds averaged diffusion equation becomes

$$\partial_t C + \underline{U} \cdot \nabla C = \nabla \cdot (\underline{\underline{D}} \cdot \nabla C)$$

which reduces to the diffusion equation only if $D_{ij} = \tilde{D}$ is not a function of $\underline{x}$. Otherwise resulting equation is not trivial.

At a basic level, one can observe that, in general

$$\partial_t C + \nabla \cdot (\underline{U}^E C) = \underline{\underline{D}}^E \nabla^2 C$$

where $U_i^E = U_i + \partial_j D_{ij}$ and $D_{ij}^E = (D_{ij} + D_{ji})/2$

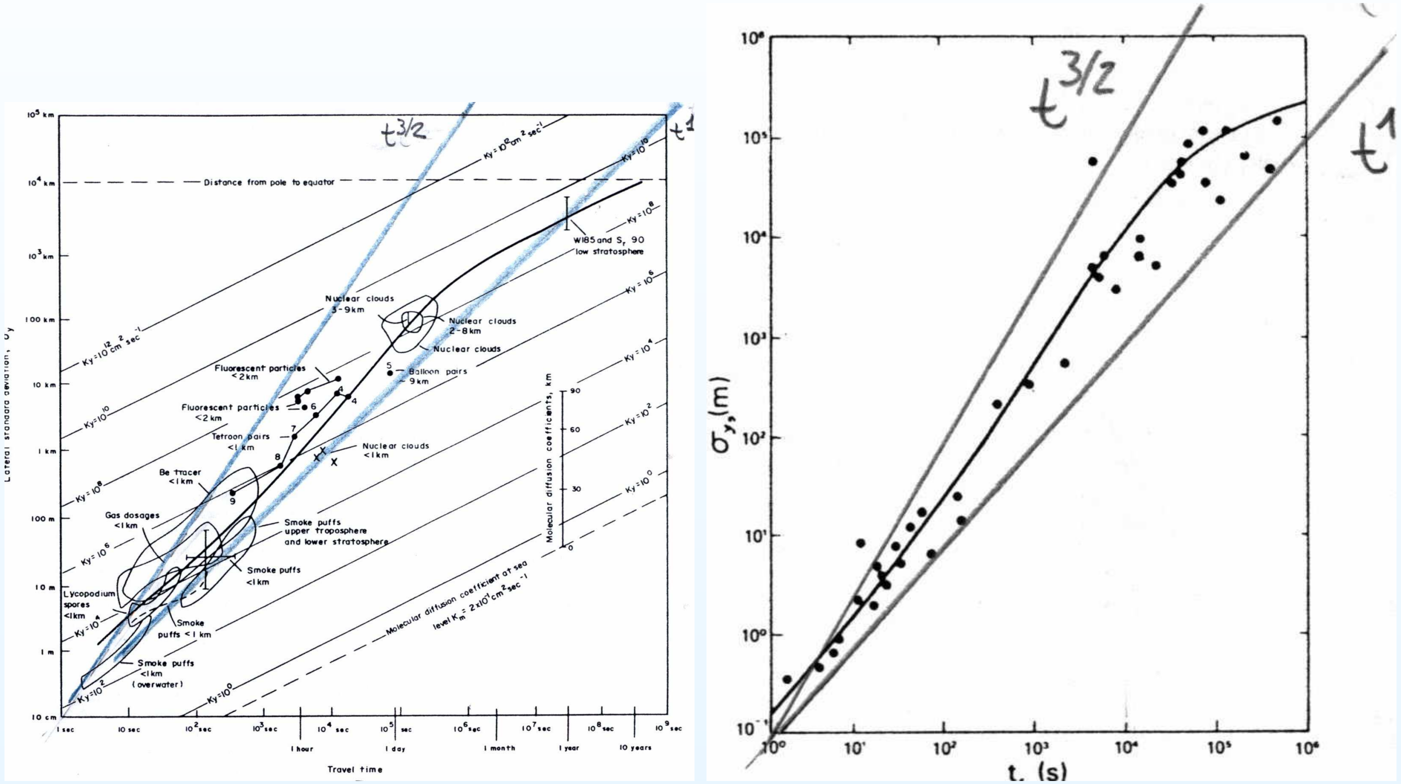
Turbulent dispersion and atmospheric energy spectrum

Observed dispersion properties strongly depend on the ratio between the observation time ΔT and the integral correlation time $T_I = \int_0^\infty R(t)\,\mathrm{d}t$.

Depending on this ratio, the energy spectrum is divided into two parts:

- scales for which correlation time is larger that Δt, that do not contribute to dispersion but can be regarded as responsible for advection of the center of mass
- scales for which the correlation time is smaller than Δt for which many realization occur during the observed phenomenon so that their effect is dispersion about the center of mass

Horizontal dispersion observations



Horizontal dispersion data (for tropospheric cases only, on the right) from Gifford (1982). Note that the standard deviation increases at an intermediate rate between t^1 and $t^{3/2}$, a signature of enhanced (Richardson) dispersion.

Absolute dispersion and meandering

a set of N particles, positions $x_i^{(n)}(t)$, $i = 1, 2, 3$, $n = 1, ..., N$.

(7) $$\langle x_i \rangle = \frac{1}{N} \sum_{n=1}^{N} x_i^{(n)}$$

Relative positions $y_i^{(n)} = x_i^{(n)} - \langle x_i \rangle$. $\langle y_i \rangle = 0$.

(8) $$\langle y_i y_j \rangle = \langle x_i x_j \rangle - \langle x_i \rangle \langle x_j \rangle$$

M sets of N particles each

(9) $$\overline{\langle x_i x_j \rangle} = \overline{\langle y_i y_j \rangle} + \frac{1}{M} \sum_{m=1}^{M} \langle x_i \rangle^{(m)} \langle x_j \rangle^{(m)} = \overline{\langle y_i y_j \rangle} + \overline{\langle x_i \rangle \langle x_j \rangle}$$

Relative dispersion and meandering

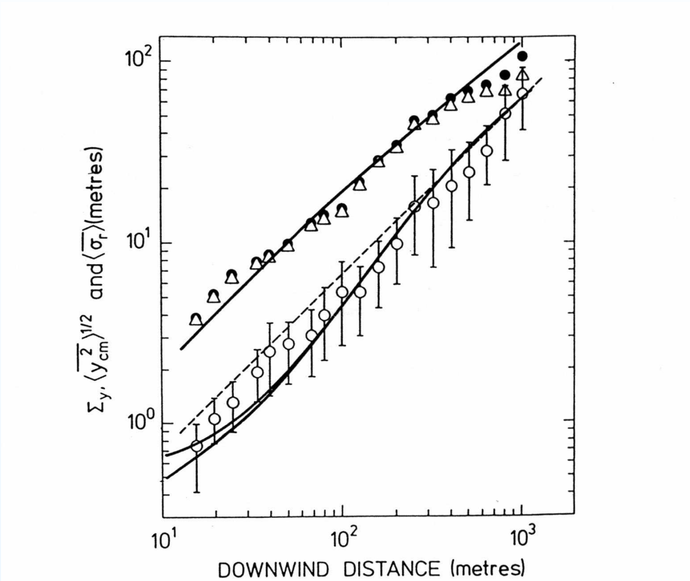
Let us consider the separation between two particles, for each m set: $\Delta x_i = x_i^{(j)} - x_i^{(l)}$.

(10) $$\langle \Delta x_i^2 \rangle \simeq \frac{1}{N^2} \sum_{j=1}^{N} \sum_{l=1}^{N} [x_i^{(j)} - x_i^{(l)}]^2 = 2\langle x_i^2 \rangle - 2\langle x_i \rangle^2$$

(11) $$\langle \Delta x_i^2 \rangle \simeq 2\langle y_i^2 \rangle$$

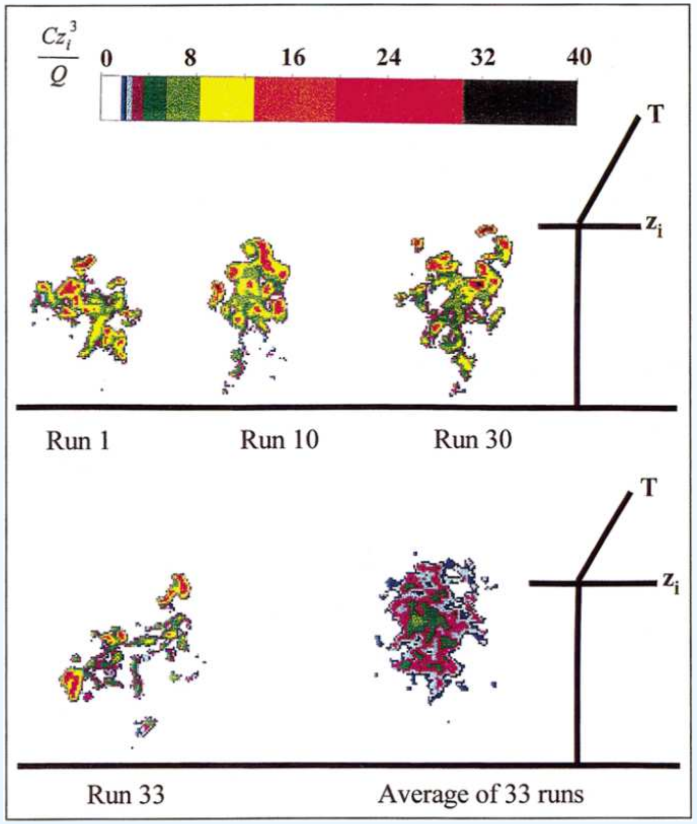
namely, the variance of the distance between pair of particles is two times the dispersion about the mean position.

Surface layer: meandering from field data



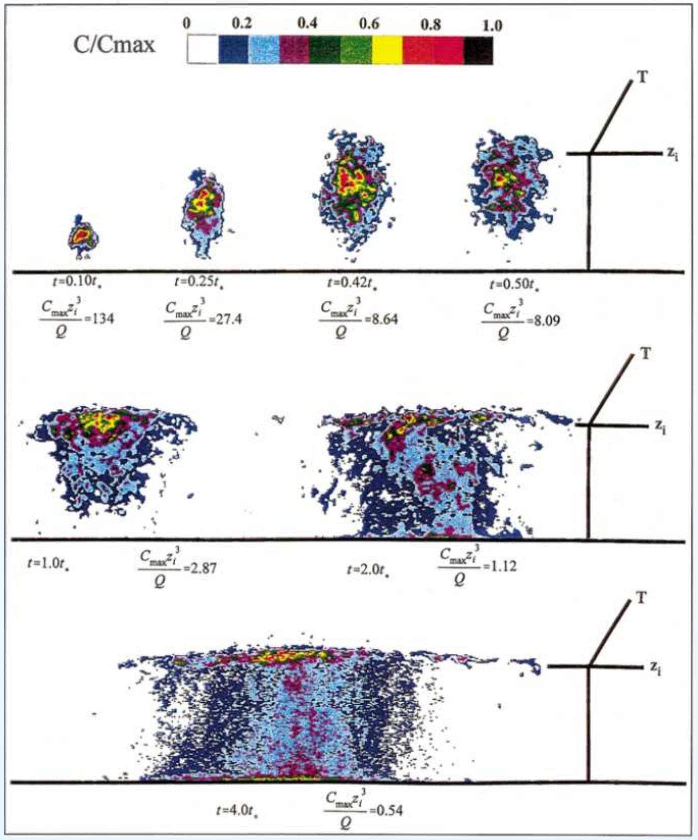
A field experiment carried out in Denmark: plume lateral meandering (Mikkelsen et al., 1987): the figure shows the instantaneous observed standard deviation about the center of mass, averaged over the ensemble of realizations (open circles) with error bars; the dispersion associated with the movement of the centreline (triangles); the total lateral dispersion (full circles).

CBL meandering: laboratory data



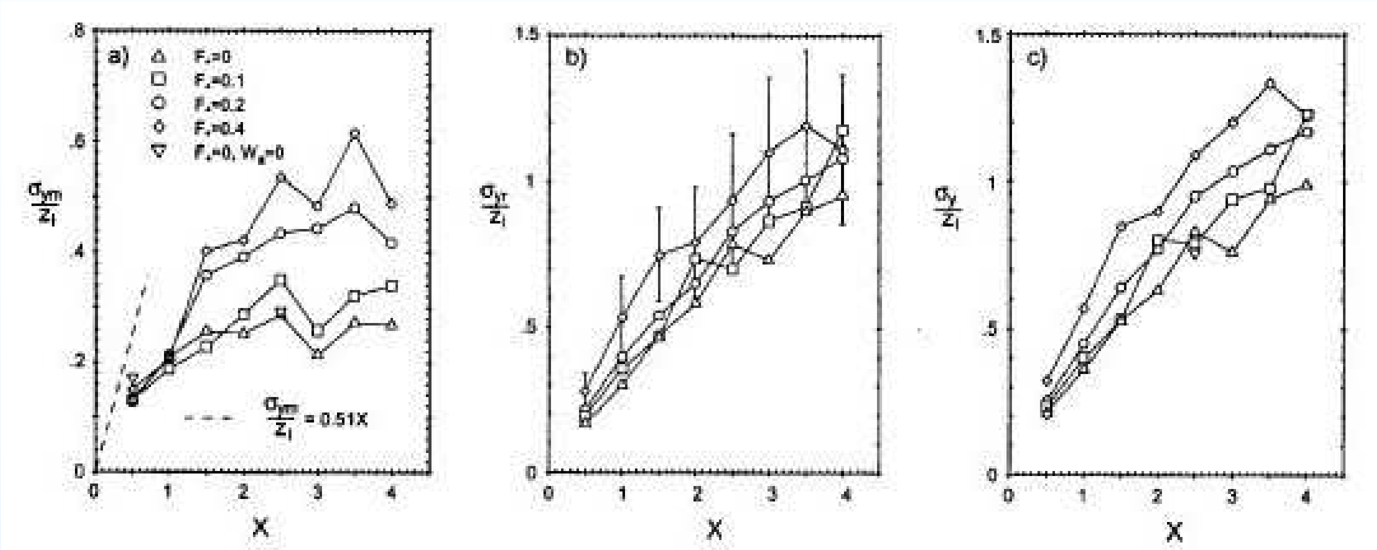
Single realizations and ensembe average for a puff released in the model CBL, after Snyder et al. (2002).

CBL meandering: laboratory data



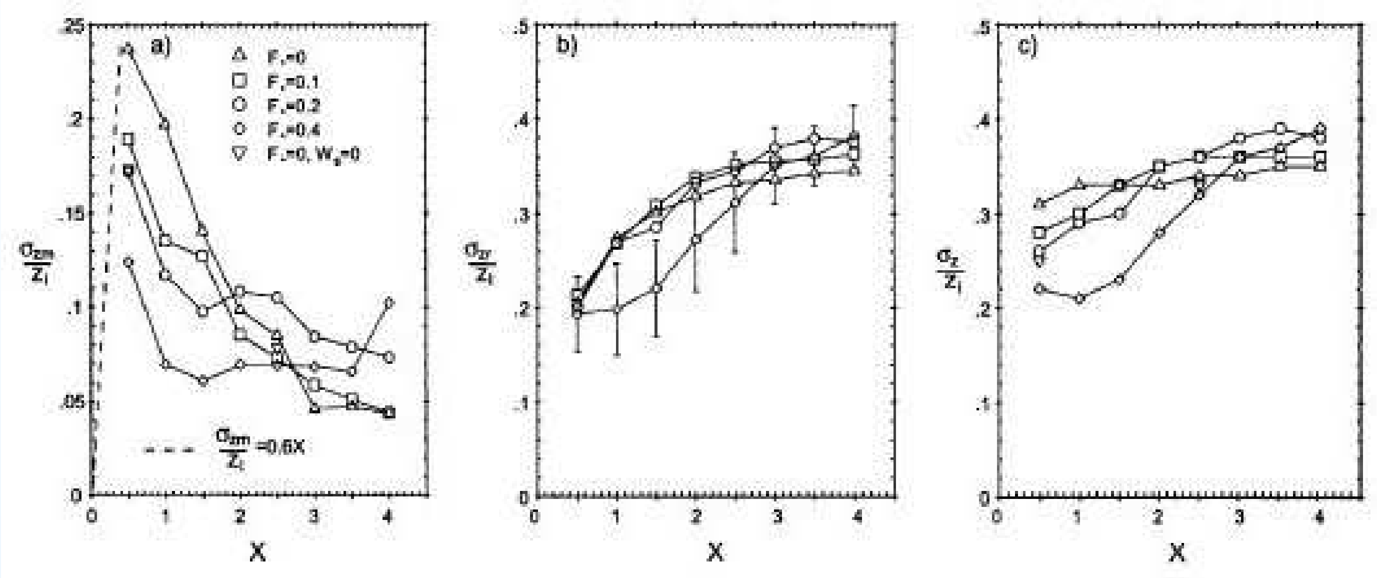
Centerplane concentration averaged over the 33 realizations, at different times, after Snyder et al. (2002).

CBL meandering: laboratory data



RMS meandering, relative and total dispersion in lateral direction for plumes with different initial buoyancy in the model CBL, after Weil et al. (2002)

CBL meandering: laboratory data



RMS meandering, relative and total dispersion in vertical direction for plumes with different initial buoyancy in the model CBL, after Weil et al. (2002)

Some evaluations of the eddy dispersion coefficient

Assuming the existence of an inertial subrange characterised by the spectrum

(12) $$E(k) = C_K \varepsilon^{2/3} k^{-5/3}$$

ε may be derived ($C_0 = 2$, $C_K = .5$) fitting the data, leading to an estimate of the eddy 'dispersion coefficient' as a function of the scale (wavenumber)

(13) $$D(k) = \frac{2\sigma^4(k)}{C_0 \varepsilon}$$

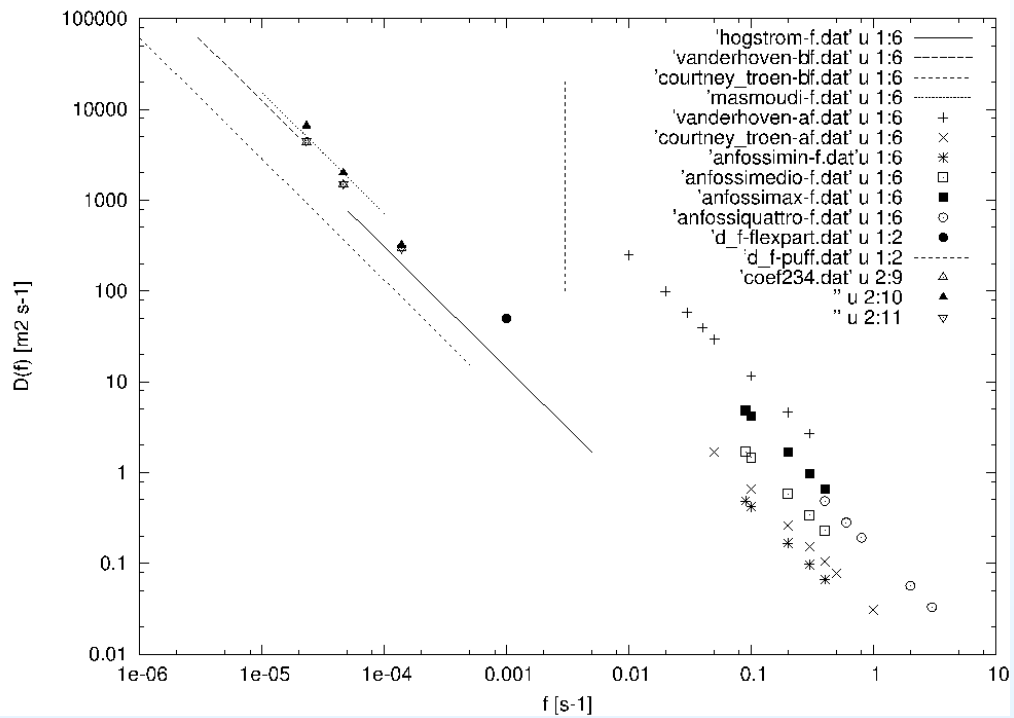
Dissipation estimates

Estimates of the dissipation ε are:

- high frequency, near surface data: $\varepsilon = 10^{-2} \div 10^{-3}\,\mathrm{m}^3\mathrm{s}^{-2}$
- low frequency, near surface data: $\varepsilon = 10^{-4}\,\mathrm{m}^3\mathrm{s}^{-2}$
- troposphere: $\varepsilon = 10^{-4}\,\mathrm{m}^3\mathrm{s}^{-2}$
- including statosphere: $\varepsilon = 10^{-5}\,\mathrm{m}^3\mathrm{s}^{-2}$

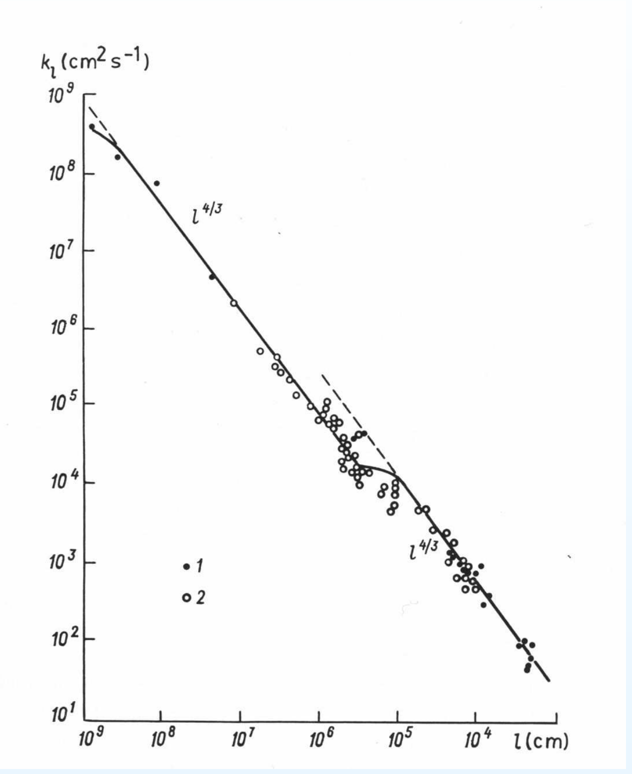
Some evaluations of the eddy dispersion coefficient

note that $\sigma^2(k) \sim k^{-2/3}$ and $D(k) \sim k^{-4/3}$



Eddy dispersion coefficient as a function of wavenumber, from different spectra.

A short cruise in the ocean



Horizontal dispersion coefficient as a function of scale, from diffusion experiments in the ocean, after Monin and Ozmidov (1985).

Other estimates of D

- surface layer data: Maryon (1998): $0.3 - 5\ 10^3 m^2 s^{-1}$
- large scale horizontal dispersion: Gifford (1982): $5\ 10^4\ m^2 s^{-1}$ for tropospheric data only, $10^5\ m^2 s^{-1}$ including stratospheric data
- long range tracer dispersion in the PBL (Sorensen et al., 1998): $6\ 10^3 m^2 s^{-1}$
- stratospheric balloons (Lacorata et al., 2004): $10^7\ m^2 s^{-1}$
- horizontal growth of Etna plume (Tiesi et al., 2006): $10^3\ m^2 s^{-1}$
- PUFF model (Searcy et al., 1998): horizontal $1 - 8\ 10^4 m^2 s^{-1}$, vertical $10^2 m^2 s^{-1}$; but using high resolution simulations $D_h = 10^2$ (Tanaka and Yamamoto, 2002).

A fundamental warning: is it correct to use this 'coefficient'?
Is it justified in terms of scale separation?

A warning on flux-gradient diffusion

Quoting Stanley Corrsin (1974):

> [. . .] a gradient transport model requires (among other things) that the characteristic scale of the transporting mechanism [. . .] must be small compared with the distance over which the mean gradient of the transported property changes appreciably
>
> [. . .] nearly all traditional turbulent transport problems violate this requirement [. . .]
>
> There has been an astonishing lack of amazement at the partial success of these models.

Fick's law limitations

- CBL: strong asymmetry, scales on transp. mech. $\simeq$ domain size $\simeq$ concentration gradient scale
- near source dispersion: $t < T_L$ (no asymptotic behaviour)
- finite size domain (Boffetta et al, 2000: Finite Size Lyapunov Exponents: statistics are better computed at fixed size rather than at fixed time)

Summary on dispersion observations

- Taylor (1921) and Richardson (1926) provide basic paradigms for absolute and relative dispersion
- dispersion can be described by advection-diffusion equation for mean concentration
- observational time is important to define the part of the spectrum that acts as diffusive (and consequently, the “meandering” problem)
- diffusion coefficient can be estimated from spectra
- there is a serious warning on flux-gradient transport model

Beyond the diffusion equation

- *Lagrangian stochastic modelling of turbulent dispersion*

The dispersion modelling problem

If velocity field $\underline{u}(\underline{x}, t)$ were known . . .

The dispersion modelling problem

If velocity field $\underline{u}(\underline{x}, t)$ were known ... we would write

$$\underline{X}(t) = \underline{X}(t_0) + \int_{t_0}^{t} \underline{V}(t')\,\mathrm{d}t'$$

where $\underline{V}(t) = \underline{u}(\underline{X}(t), t)$ (implicitly Re$>> 10^3$ to neglect molecular effects and consider marked fluid parcels as passive tracer)

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and the problem of “dispersion modelling” would disappear!

However, $\underline{u}(\underline{x}, t)$ is hardly available. Here we concentrate on the case in which it is partially known through some “properties”.

Turbulent velocity

Full knowledge of the velocity field only comes from DNS for very special cases (and for small Re).

Other possibilities are related with uncertainties:

- Large Eddy simulations: $\underline{u}(\underline{x}, t)$ is spatially filtered $\longrightarrow$ unresolved scales have to be considered
- Reynolds Averaged Navier-Stokes equations: only first few moments $\langle u_i(\underline{x}, t)\rangle$, $\langle u_i(\underline{x}, t) u_j(\underline{x}, t)\rangle$ are known
- Primitive Equations: truncation errors
- some Similarity Theory: e.g., in the surface layer
- measurements: hopefully some reliable statistics but very spatialy/temporally limited

in any case $\underline{X}(t)$ cannot be computed with further assumptions.

Stochastic approach: a first attempt

The fisrt model we can write is:

$$\mathrm{d}X_i = U_i(\underline{X}, t)\,\mathrm{d}t + B_{ij}\,\mathrm{d}W_j$$

where $\mathrm{d}W$ is a Wiener process:

$$\langle\,\mathrm{d}W_i(t)\rangle = 0\ ,\ \langle\,\mathrm{d}W_i(t)\,\mathrm{d}W_j(s)\rangle = \delta_{ij}\delta(t-s)\,\mathrm{d}s\,\mathrm{d}t$$

This is not directly a model for the velocity but still is a model for dispersion with an assumption: ???

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This is not directly a model for the velocity but still is a model for dispersion with an assumption:
the stochastc process X is a Markov process, *i.e.*, the velocity fluctuations u' with respect to U, are delta correlated in time

$$\langle u'(s)u'(t)\rangle \sim \delta(s-t)$$

SDE FPE equivalence

In general, given a Stochastic Differental Equation

$$\mathrm{d}y = a(y,t)\,\mathrm{d}t + b(y,t)\,\mathrm{d}W$$

the Probability Density Function of y is described by the Fokker-Planck Equation

$$\partial_t p(y;t) + \partial_y [a(y,t)p(y;t)] = \frac{1}{2}\partial_y^2 [b^2 p(y;t)]$$

Diffusion equation

Given that, it is straightforward to identify our “first attempt” with the diffusion equation, and so

$$B_{ij} \equiv \sqrt{2D_{ij}}$$

with

$$D_{ij} = \lim_{t \to \infty} \frac{\overline{x_i x_j}}{2t} = \overline{u_i' u_j'} T_{ij}$$

We are still dealing with a diffusion process (we made no progress!!!). But what is so wrong in modelling our problem with a diffusion equation? Remember Corrsin.

A further step: first-order Markov

LSM based on Markovianity assumption on $(\mathbf{x}, \mathbf{u})$
(implies equivalence between) stochastic differential or Langevin equation (LE)

$$\mathrm{d}u_i = a_i \,\mathrm{d}t + b_{ij}\,\mathrm{d}W_j$$

$$\mathrm{d}x_i = u_i \,\mathrm{d}t$$

Fokker-Plank equation (FPE)

$$\partial_t p + \partial_{x_i}(u_i p) + \partial_{u_i}(a_i p) = \frac{1}{2}\partial_{u_i}\partial_{u_j} b_{ij} p$$

Some nice property: no flux-gradient is implied

For the Markov process (x, u),

$$\frac{\partial n}{\partial t} = -\frac{\partial}{\partial x_i}(v_i n) - \frac{\partial}{\partial v_i}(a_i n) + \frac{C_0 \varepsilon}{2} \frac{\partial^2 n}{\partial v_i \partial v_i} \tag{14}$$

To get an equation for $\overline{c}$ integrate term by term over $\mathrm{d}^3\mathbf{v}$.
As $n = 0$ and $\partial n / \partial \nu = 0$ at the boundary of the integration volume, we can use Gauss and Green theorems to reduce Eq. 14 to

$$\frac{\partial \overline{c}}{\partial t} = -\frac{\partial}{\partial x_i} \int \mathrm{d}^3\mathbf{v}\,(v_i n) \tag{15}$$

If $v_i = \overline{u}_i + v_i'$:

$$\frac{\partial \overline{c}}{\partial t} = -\overline{u}_i \frac{\partial \overline{c}}{\partial x_i} - \frac{\partial}{\partial x_i} \int \mathrm{d}^3\mathbf{v}'\,(v_i' n) \tag{16}$$

The integral in the last term is identified as the turbulent flux.

Small scale consistency

To the first order, the structure function of an SDE is

$$\langle (u_i(t) - u_j(t_0))^2 \rangle = b_{ij}^2 (t - t_0)$$

According to K41, Lagrangian structure function of turbulent velocity within the inertial subrange is

$$\langle (u_i(t) - u_j(t_0))^2 \rangle = C_0 \varepsilon (t - t_0) \delta_{ij}$$

so that

$$b_{ij} = C_0 \varepsilon \delta_{ij}$$

In particular, for local isotropy $b_{ij}^2 \to b^2$

Eulerian statistical consistency

"Well-mixed" condition (Thomson, 1987)

$$\text{if } p(\mathbf{x}, \mathbf{u}, t_0) = p_\mathsf{E}(\mathbf{u}) \, \text{unif}_\mathcal{D}(\mathbf{x})$$
$$\text{then } p(\mathbf{x}, \mathbf{u}, t) = p_\mathsf{E}(\mathbf{u}) \, \text{unif}_\mathcal{D}(\mathbf{x}) \text{ for } t > t_0$$

Assuming $b_{ij}^2 = C_0 \varepsilon \delta_{ij}$ (K41), the solution for a_i is given by

$$a_i = \frac{C_0 \varepsilon}{2} \frac{1}{p_\mathsf{E}} \frac{\partial p_\mathsf{E}}{\partial u_i} + \frac{\phi_i}{p_\mathsf{E}}$$

$$\frac{\partial \phi_i}{\partial u_i} = -\frac{\partial p_\mathsf{E}}{\partial t} - u_k \frac{\partial p_\mathsf{E}}{\partial x_k}$$

two-particle LSM: relative dispersion

If that state is identified by the variable $(\underline{x}^{(1)}, \underline{x}^{(2)}, \underline{u}^{(1)}, \underline{u}^{(2)})$ where ^{i} identify the i-th particle ($i = 1, 2$), it is straightforward to extend the WMC concept to a two particle system, provided that P_E is the Eulerian two-point pdf (Thomson, 1990).

The remarkable difference is that the two point Eulerian pdf must contain the spatial structure of turbulence via the particle distance $\Delta_i x = ||\underline{x}^{(1)} - \underline{x}^{(2)}||$.

In particulare it must be

$$\langle (\underline{u(x_i^{(1)})} - \underline{u(x_i^{(1)})})^2 \rangle = C_K (\varepsilon \Delta_i)^{2/3}$$

Focus on the open problem of non-uniqueness

The solution of the WMC problem is not unique. Any function f with $\partial_u f = 0$ can be added to ϕ without altering the well-mixedness.
Non-uniqueness of the solution of the WMC problem open the possibility to introduce furhter information on the flow structure.

In fact, so far we only considered one-point Eulerian PDF that cannot account for spatial structure.

This nonuniqueness can manifest itself as a “spin” term, that induces a mean rotation of the Lagrangian turbulent velocity vector (Borgas et al., 1997; Sawford, 1999).

Determination of b

The dissipation rate of turbulent kinetic energy $\varepsilon = \nu \left(\overline{\partial_{x_i} u_j \; \partial_{x_j} u_i} \right)$ is hardly accessible to direct measurements except in few cases.
The Kolmogorov energy spectrum

$$E(k) = C_K \varepsilon^{2/3} k^{-5/3}$$

provides a mean for indirect ε estimation.

If C_K is known (commonly accepted value around 2), and measurements of the energy spectrum are available, ε can be retrieved by fitting the above equation on data.

Determination of b: K41 energy spectrum: caveats

spectra measured at a fixed point (Eulerian spectra in frequency domain) in this case there is the need to make some assumptions: in principle Taylor frozen turbulence must apply; however a relationship not based on the mean wind can be used to move from frequency to wavenumber domain, which involves the ratio of Eulerian to Lagrangian scales $\beta = C_K^{2/3}/(\sqrt{2}C_0)$ to transform local $\tau_\ell(\nu)$ to $\tau_\ell(k)$.

spatial temporal validity shown (and available) spectra are derived from "climatological" measurements (long records at a fixed point, mainly for measurements in the BL) or from averaging over large volumes/time (for spectra resulting from aircraft measurements). This contrasts to determinations of ε from similarity theory in, e.g., the neutral surface layer where $\varepsilon = u_*^3/(\kappa z)$ which strongly depends on "local" flow conditions.

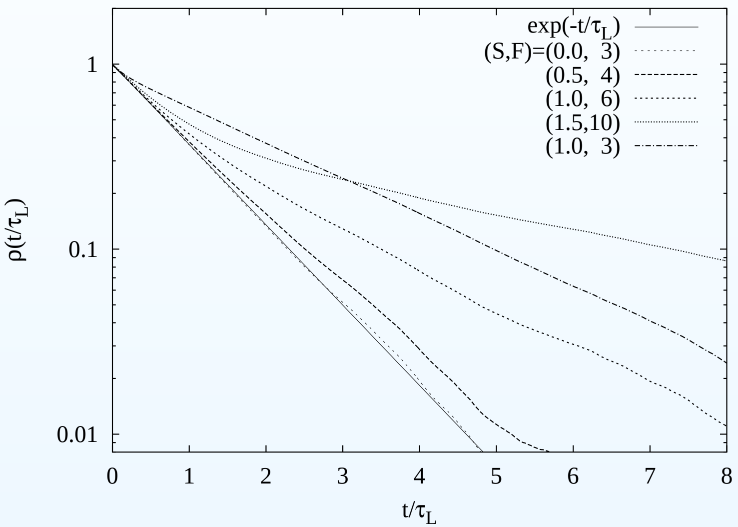
Determination of b: Lagrangian correlation time

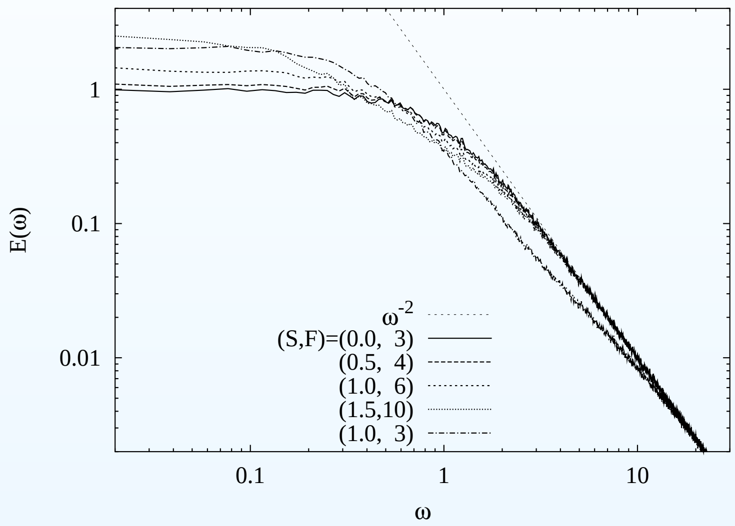
Another way of determining b is using Lagrangian properties and the relationship

$$C_0 \varepsilon = \frac{2\overline{u'^2}}{\tau_L}$$

where τ_L is a local measure of decay of the Lagrangian correlation function.
This can be used when Lagrangian measures are accessible directly (e.g., oceanic floats, but also Particle Tracking Velocimetry in lab) or when estimations of τ_L can be made available indirectly.

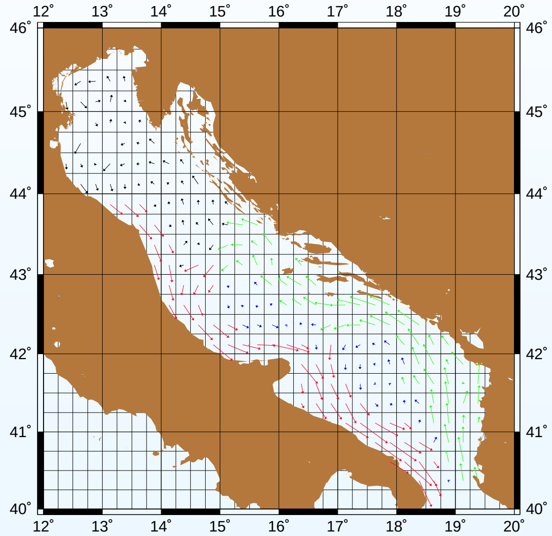
Determination of b: Lagrangian correlation time

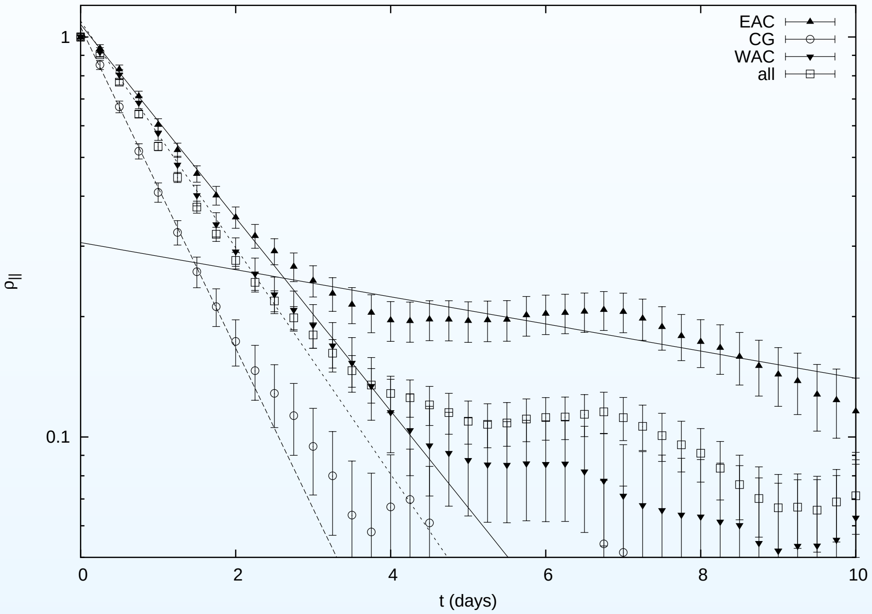




1D homogeneous stationary turbulence model with different values of third- and fourth-order moments of velocity (Maurizi and Lorenzani, 2001). Here, despite homogeneity, $T_L \neq \tau_L$.

Determination of b: Lagrangian correlation time





2D strongly non-homogeneous fields (Maurizi et al., 2004a): surface circulation as derived from drifter measurements in the Adriatic sea. Here τ_L can be defined if one looks at small time slope of $\log R_L(t)$. T_L does not necessarily exist (although in this case it does and $T_L = 2.0$–2.7 days while $\tau_L = 1.3$–1.4 days for EAC and WAC.)

Determination of b: conclusions

Summary of b determination:

- well theoretically founded
- ε can be estimated from spectra or time Lagrangian scales
- requires $k^{-5/3}$ energy spectrum
- time scale represent a “local” decorrelation time scale (not integral)

Determination of the “drift coefficient” a

consistency with Eulerian (one-point) statistics:

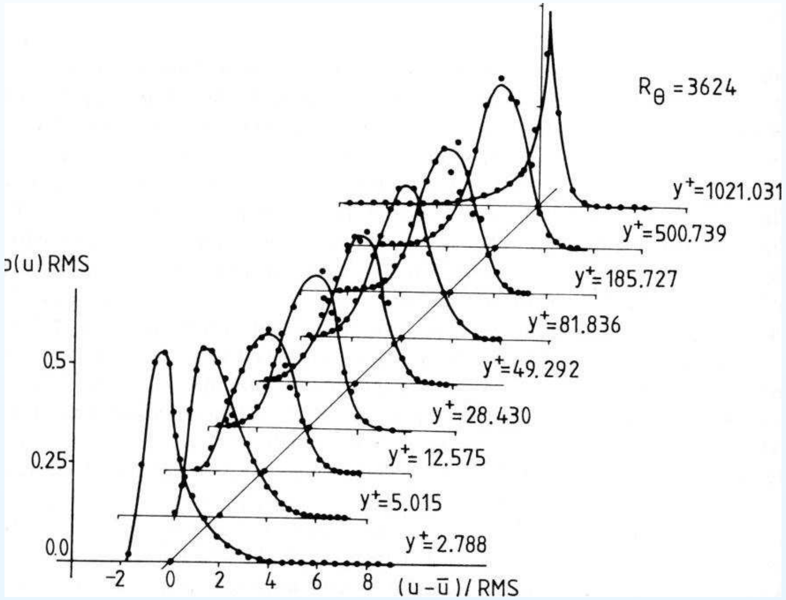
$$a_i = \frac{C_0 \varepsilon}{2} \frac{1}{p_{\mathsf{E}}} \frac{\partial p_{\mathsf{E}}}{\partial u_i} + \frac{\phi_i}{p_{\mathsf{E}}}$$

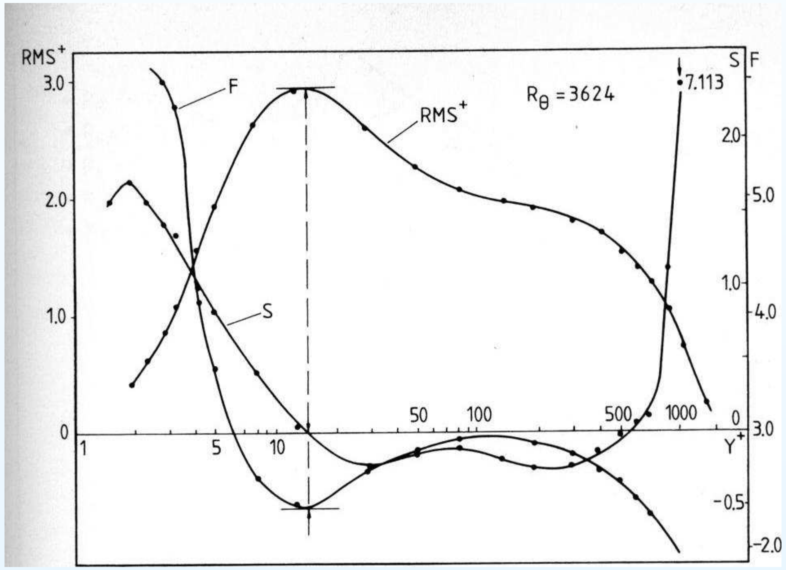
$$\frac{\partial \phi_i}{\partial u_i} = -\frac{\partial p_{\mathsf{E}}}{\partial t} - u_k \frac{\partial p_{\mathsf{E}}}{\partial x_k}$$

Determination of α: what PDF? Is turbulence Gaussian?

No, definitely not. Not even at a first level of approximation.

Some examples:





Boundary layer turbulence: varying parameter: distance from the surface (laboratory data; (Durst et al., 1987)).

Determination of α: what PDF? Is turbulence Gaussian?

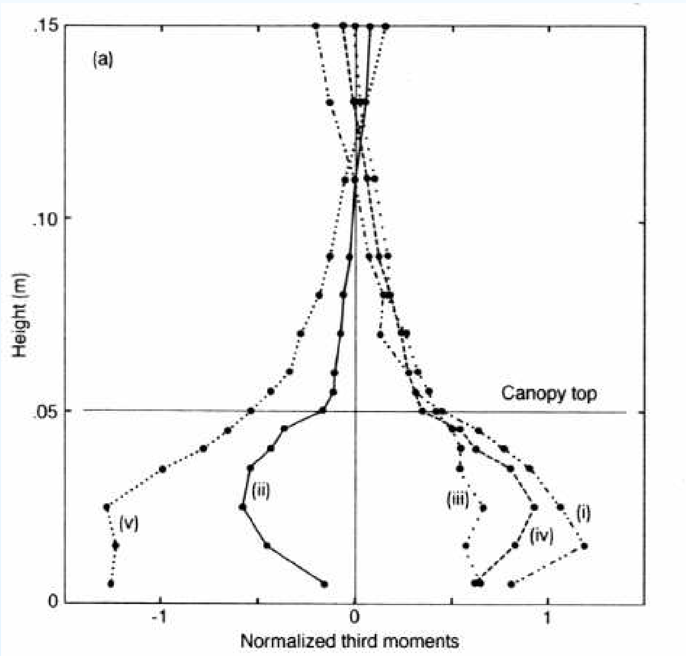
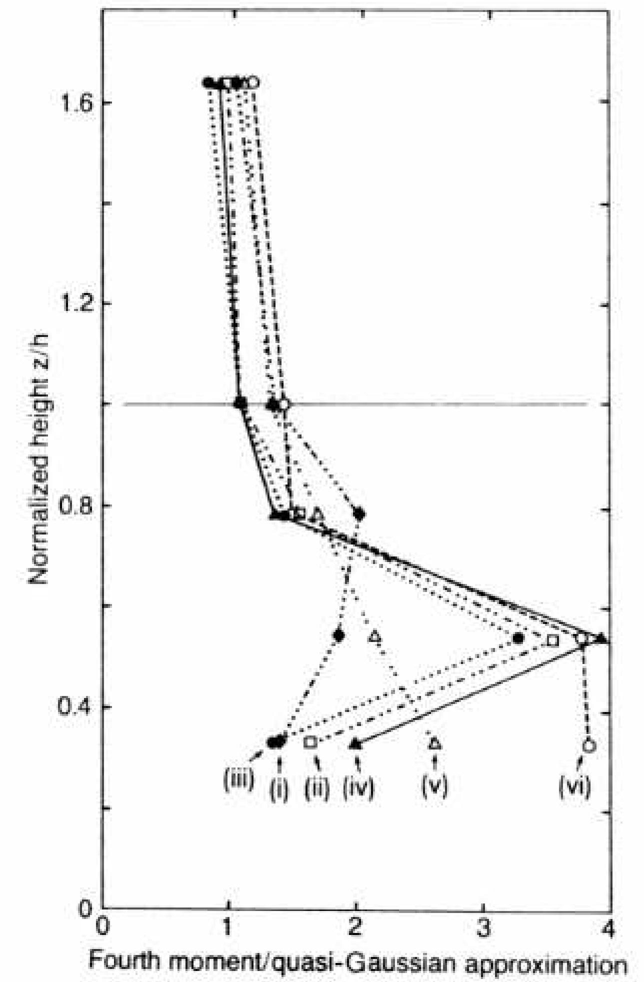
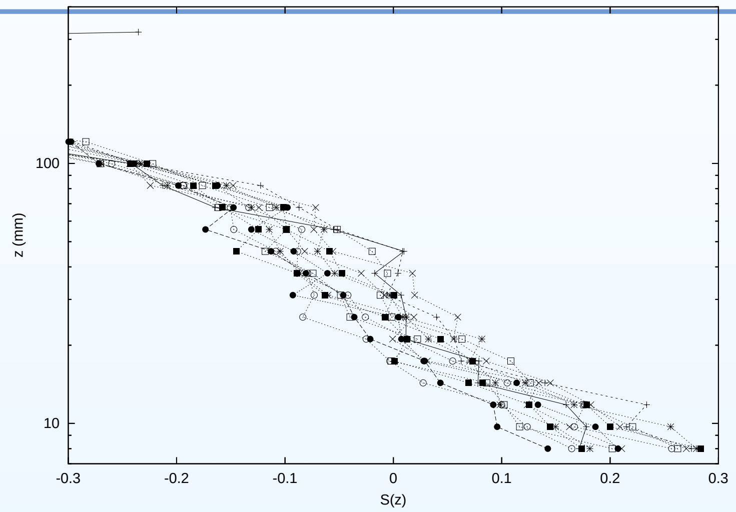


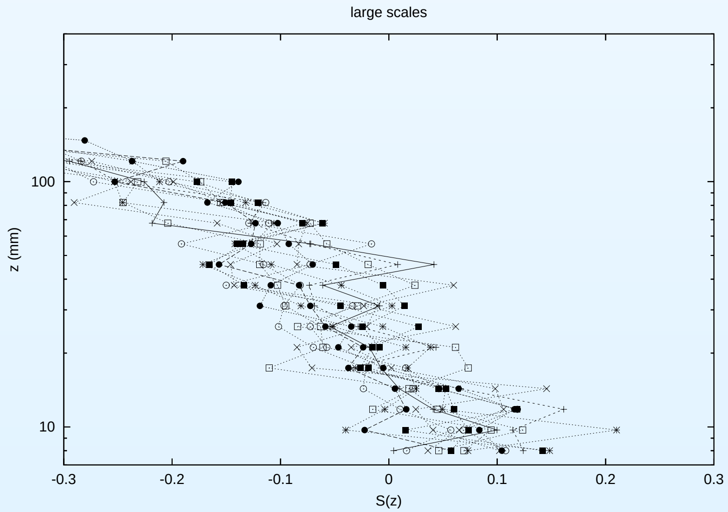
Fig. 2a.

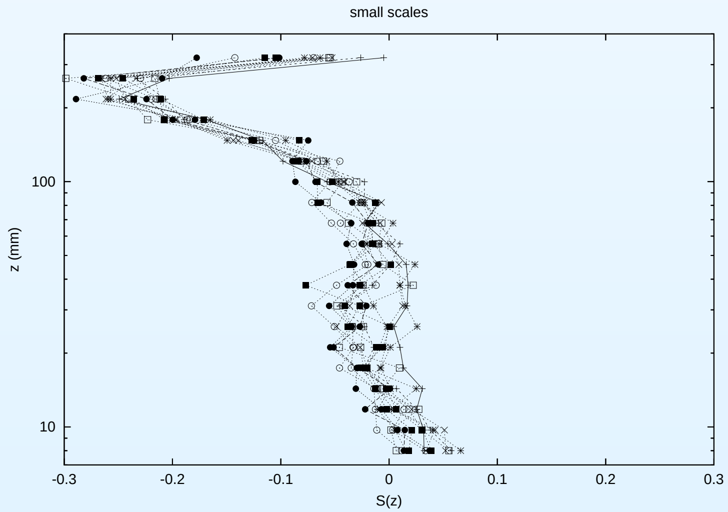


Turbulent flow in a canopy (lab. exp.).

Determination of a: what PDF? Is turbulence Gaussian?







neutral boundary layer: effect of large and small scales

Determination of α: PDF statistical moments

From experimental data we can compute moments up to some order N usually very small:

$$\mu_k^{(i)} \equiv \langle u_i^k \rangle = \frac{1}{M} \sum_{j=1}^{M} (u_j^{(i)} - \mu_k^{(i)})^k$$

for $k = 0, 1, \cdots N$ ($N < 5$?). The higher the order k the larger is the sample required for statistical significance. The error on the k-order moment can be roughly estimated by

$$\sigma_k = \alpha_k \tau / (\Delta T)$$

where α_k is a coeff dependent on k ($\alpha_2 = 6$, $\alpha_3 = 34, \ldots$), ΔT is the sampling time and τ is the correlation time of the series.

Determination of a: PDF statistical moments

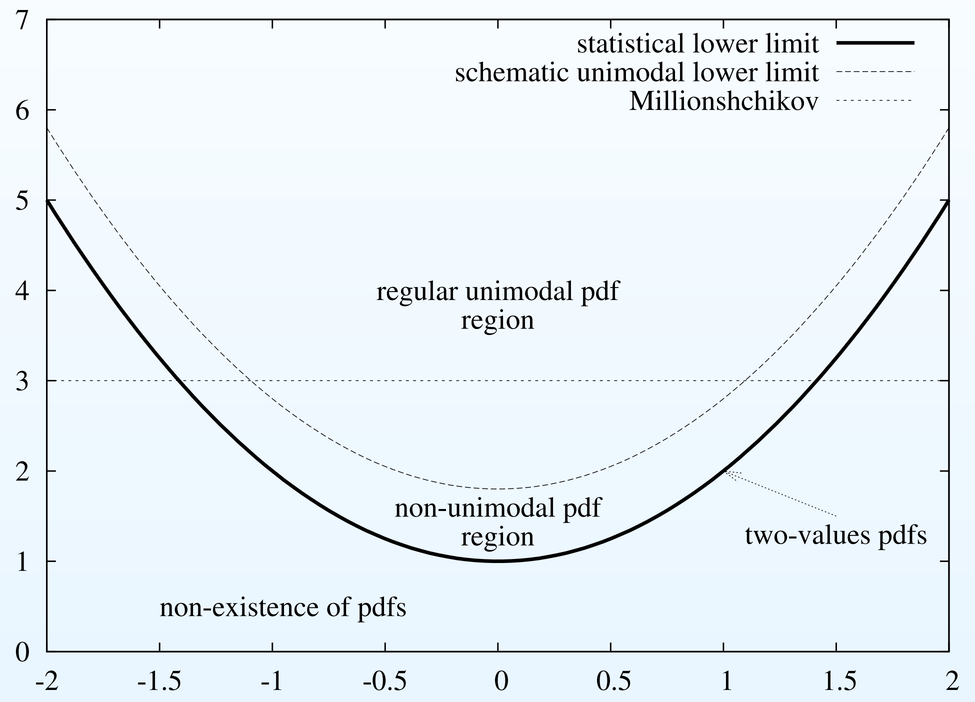
If, for a centred univariate, we limit to $k_{\text{max}} = 4$, and we normalise moments for $k > 1$ with μ_2, we obtain

$$\begin{aligned} \mu_0 &= 1 \\ \mu_1 &= 0 \\ \mu_2 &= 1 \\ \mu_3 &= S \\ \mu_4 &= K \end{aligned}$$

- S is a measure of asymmetry
- K gives a measure of how much tails are higher than for Gaussian distribution

Determination of a: properties of moments space

The $S - K$ parameter space is not $\mathbb{R} \times \mathbb{R}^+$.

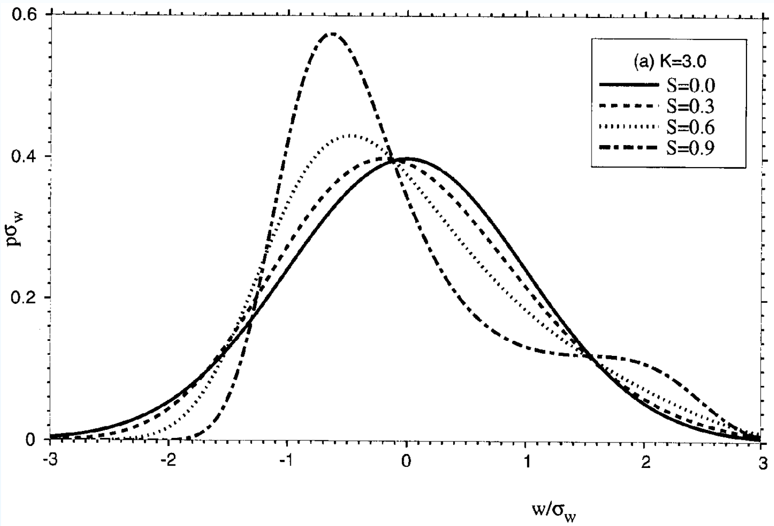


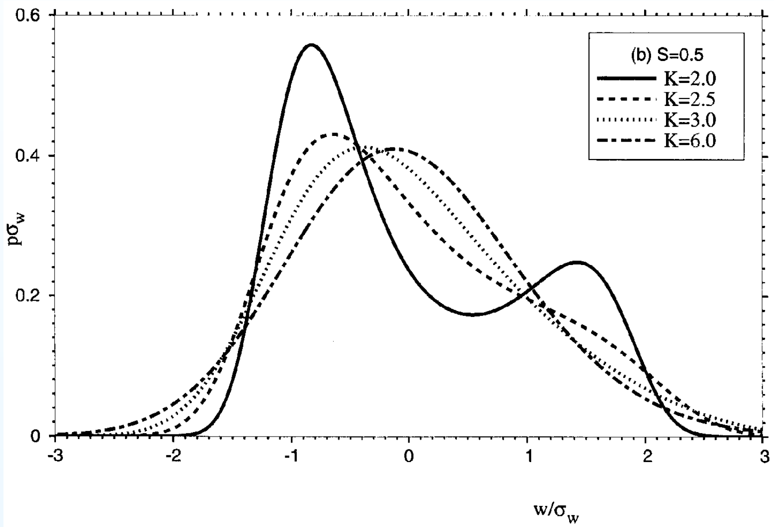
(Representation of some disregarded results from pure statistical theory)

How data are placed in the $S - K$ space?

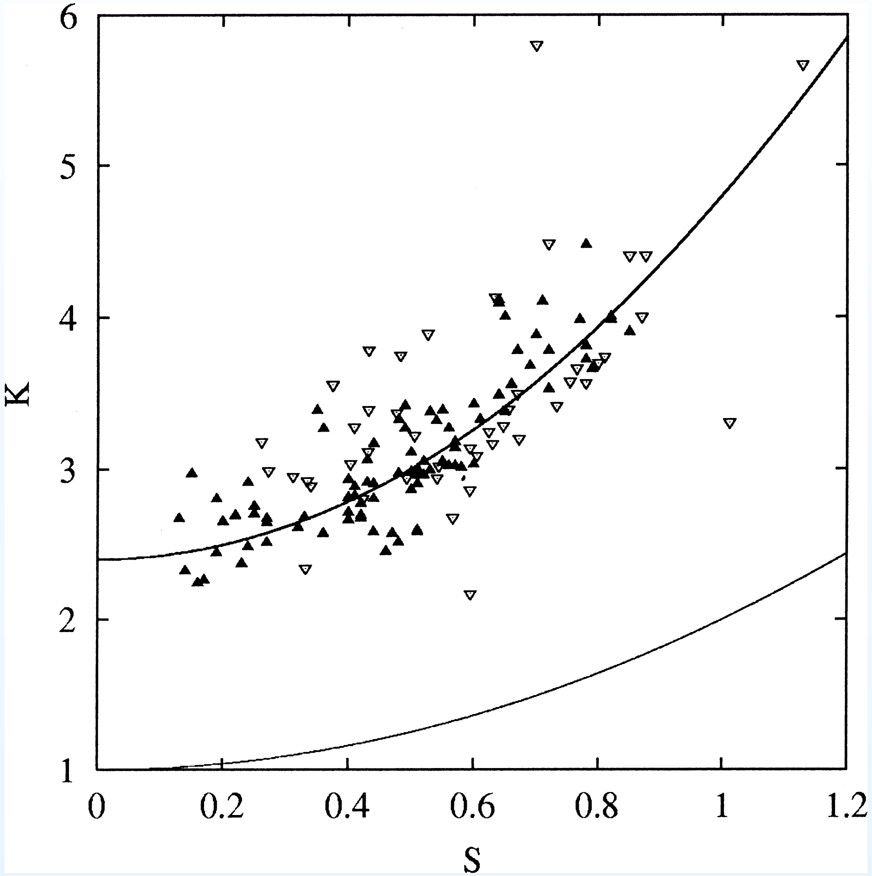
Determination of a: properties of moments space

from uni- to bi-modality



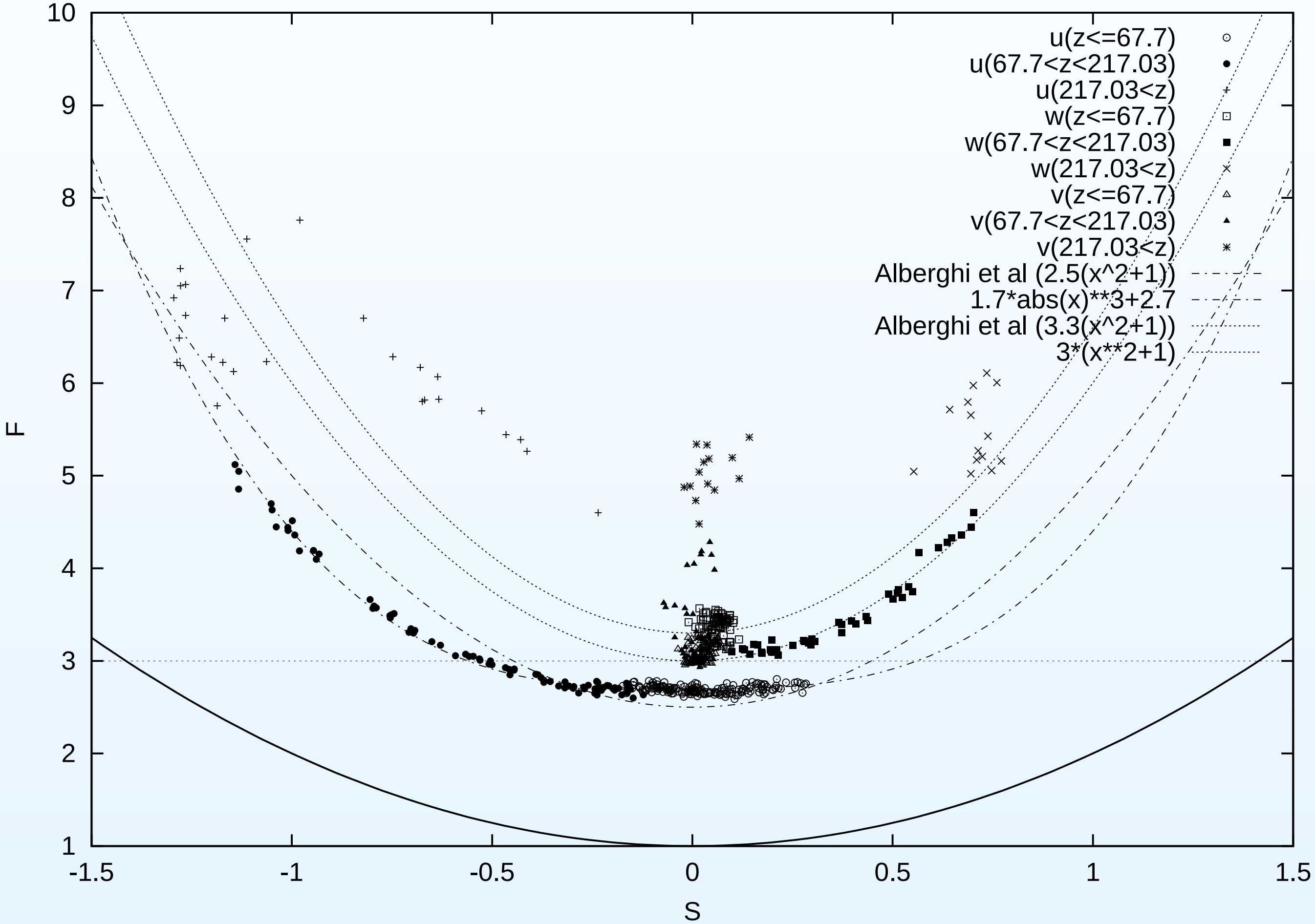


Determination of a: properties of moments space



Skewness and Kurtosis in the Convective Boundary Layer (Alberghi et al., 2002)

Determination of a: properties of moments space



Skewness and Kurtosis in the Neutral Boundary Layer (wind tunnel, Maurizi, European Turbulence Conference 11)

Determination of α: revised Millionshchikov hypothesis

If we think of the Quasi-Normal assumption as the zero-order approximation if the $S-K$ space were “linear”, we can formulate an alternative zero-order assumption for 4th order moments based on the knowledge of the $S-K$ space curvature.

$$K = 3(S^2 + 1)$$

or, going to the first-order approximation

$$K = \alpha(S^2 + 1)$$

where α can be let to include some dynamical information.
Considering data from different experimental sets, α is found in the range 2.4 – 2.6.

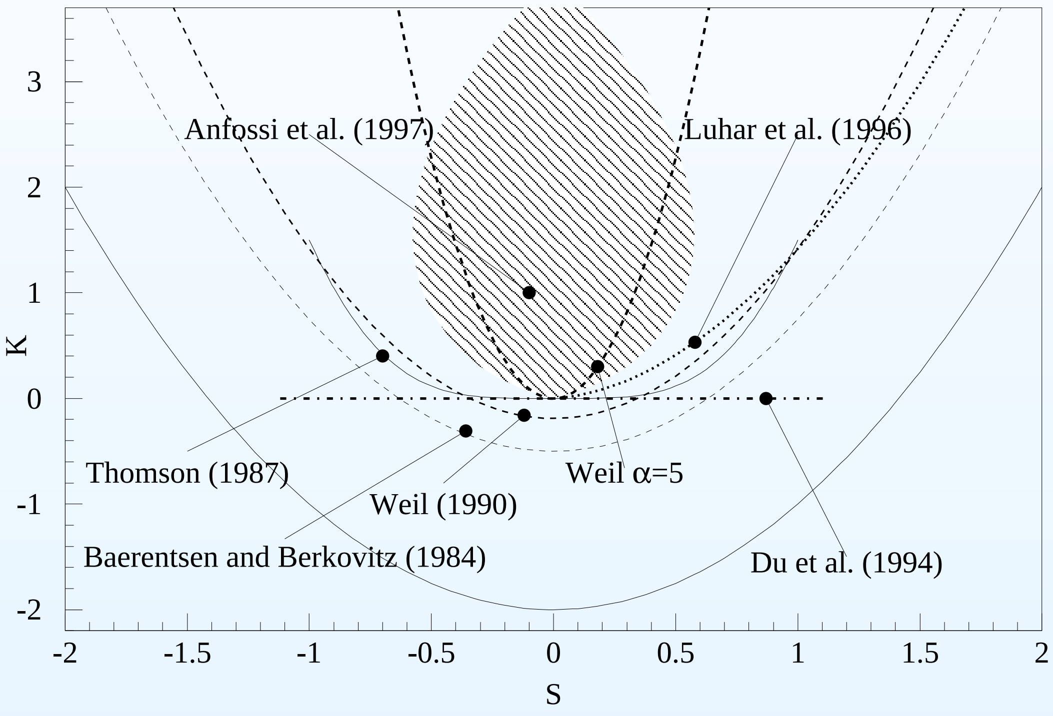
Determination of a: analytical PDF

$N = 2$: Gaussian PDF: O-U process: analytical results for the stochastic process
$N > 2$: some analytical form other that Gaussian must be used: linear combination of two normal (Gaussian) distributions has been used

$$p(u) = a_1 G(u; m_1, s_1) + a_2 G(u; m_2, s_2)$$

- six free parameters (vs. 2 parameters of a single Gaussian)
- requires moments up to the fifth: too much, our “experimentalist” can hardly afford this
- additional assumptions are needed“
  - a_i proportional to up- down-draft areas in CBL
  - $s_1 = s_2$
  - $\ldots$
  - any additional assumption is ”just an assumption“

Determination of α: bi-Gaussian solutions



bi-Gaussian solutions (+ Grahm-Charlier: shaded) (Maurizi and Tampieri, 1999).

Determination of a: Maximum Information Entropy

- based on pure statistical inference
- makes the “least” additional assumption (introduces the least bias)
- maximisation of

$$H[p(u)] = -\int p(u) \log p(u) \, \mathrm{d}u$$

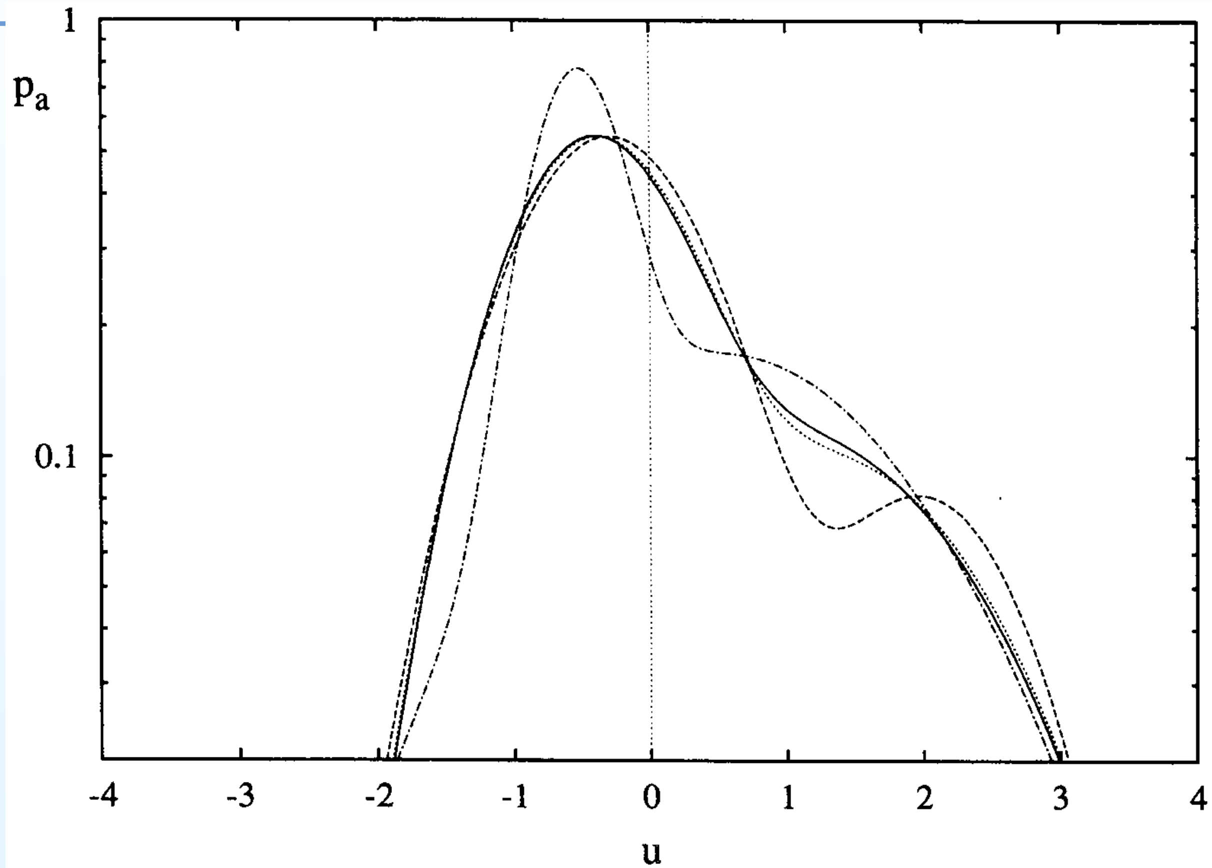
- if available information are the first N (with N even) moments of a PDF

$$p(u) = \exp(-\sum_k \lambda_k u^k)$$

- λ_k determined from

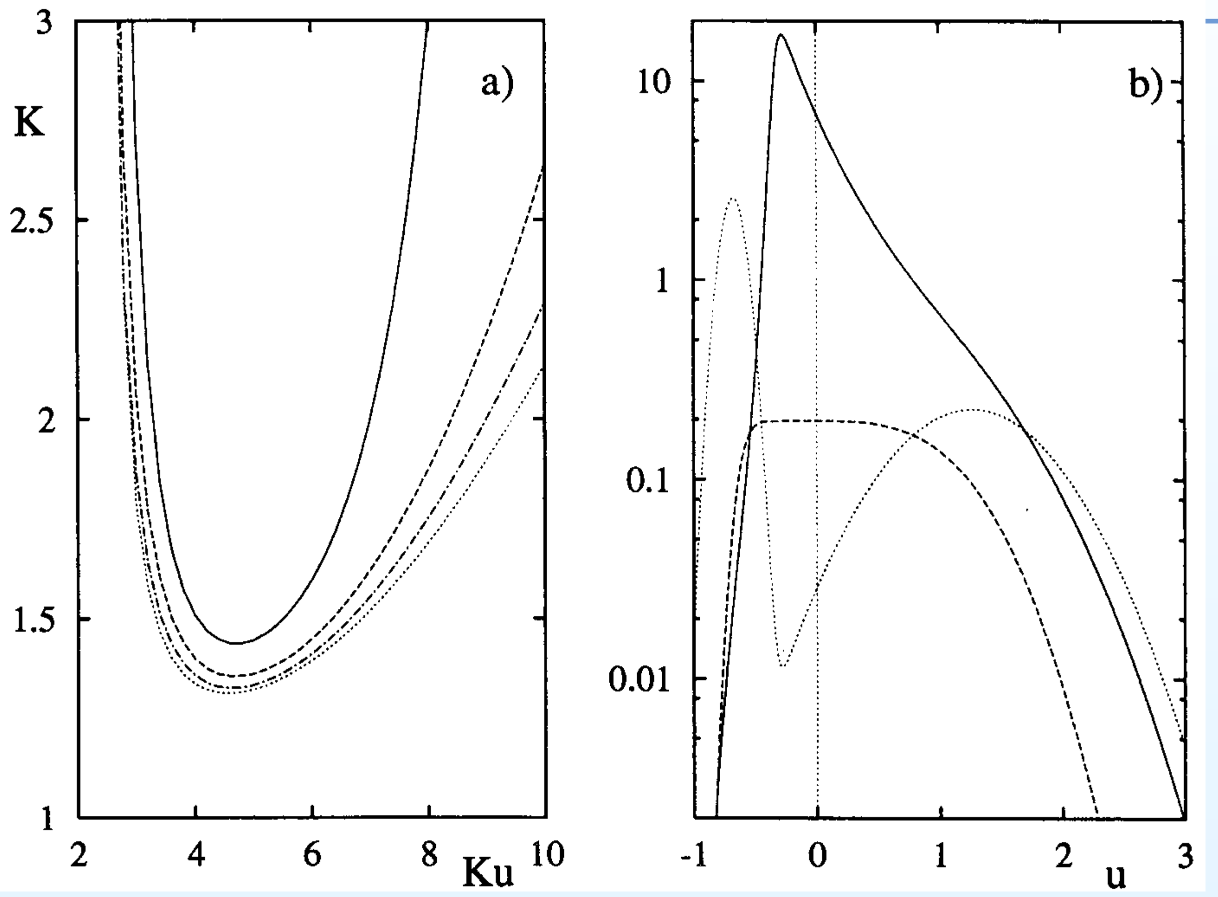
$$\int u^k p(u) \, \mathrm{d}u = \mu_k$$

Determination of a: bi-Gaussian solutions + max. entropy



bi-Gaussian PDF with given $S = 1$, $K = 4$ and for different values of the free parameter. Solid line is the one determined with maximum entropy criterion (Maurizi and Lorenzani, 2000).

Determination of a: dispersion properties of different PDFs models

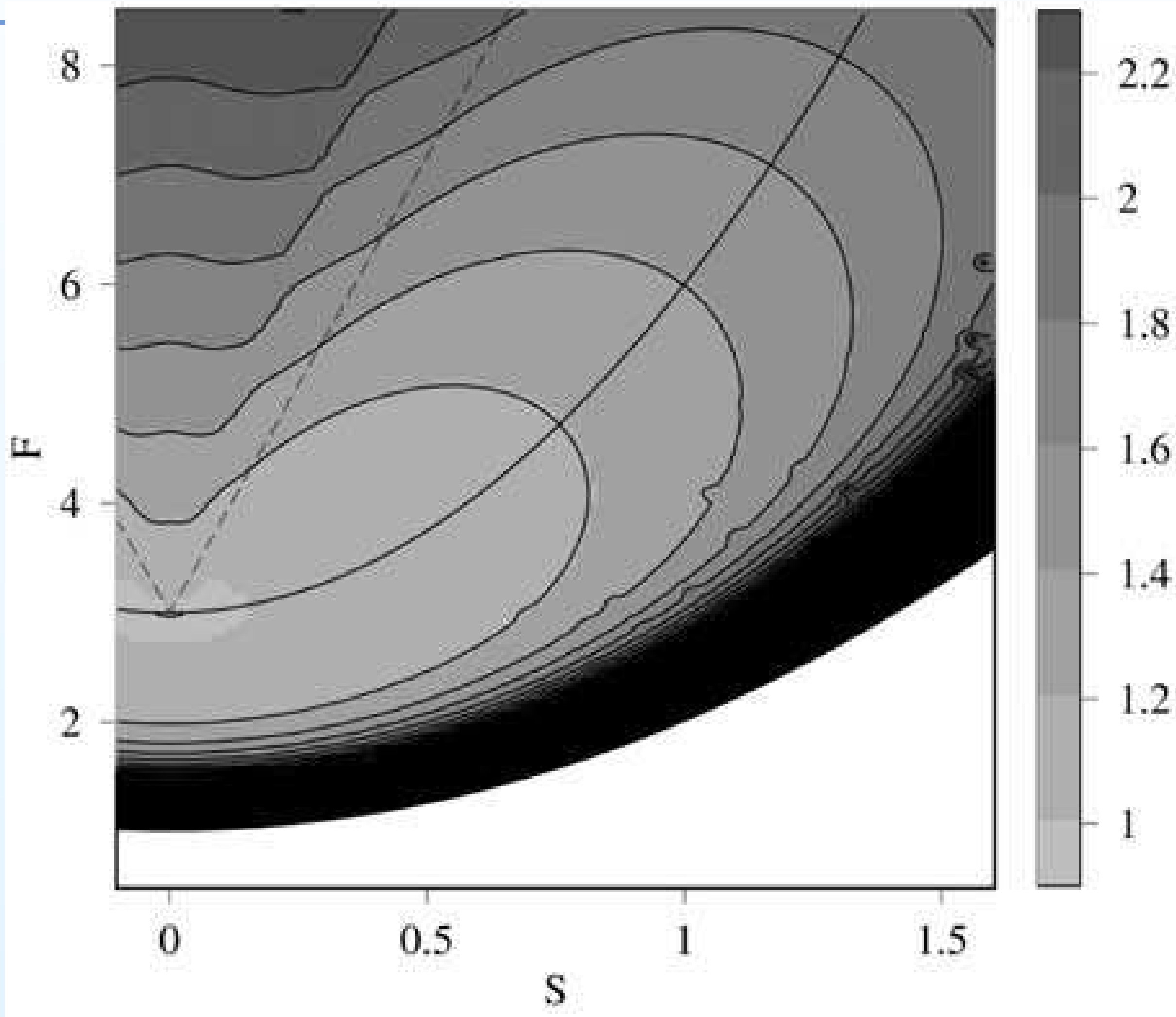


(Maurizi and Lorenzani, 2000)

After T87:

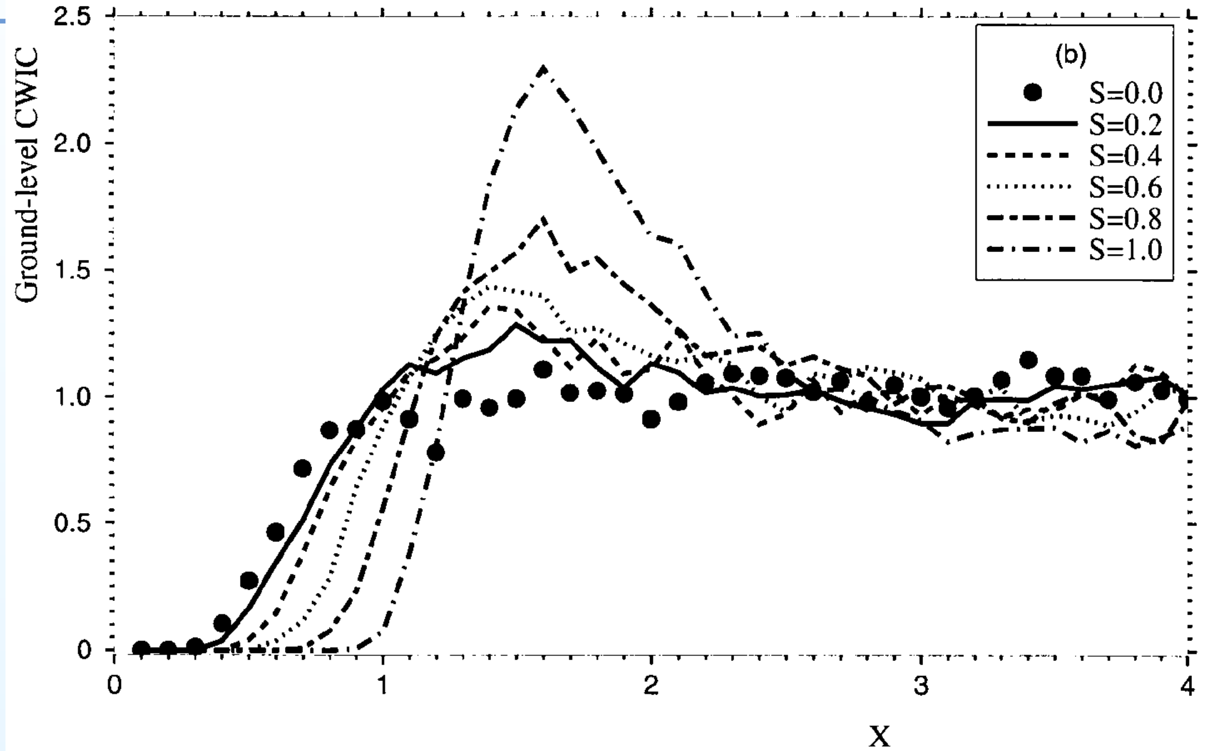
diffusion coefficient $D = \frac{1}{b^2} \int_{-\infty}^{\infty} \frac{q(u)^2}{p(u)} \, \mathrm{d}u \,, \qquad q(u) = \int_{-\infty}^{u} v p(v) \, \mathrm{d}v$

Determination of a: dispersion properties as a function of S and K



Lagrangian integral time T_L for varying S and K. (Maurizi and Lorenzani, 2001)

Determination of a: dispersion properties as a function of S and K



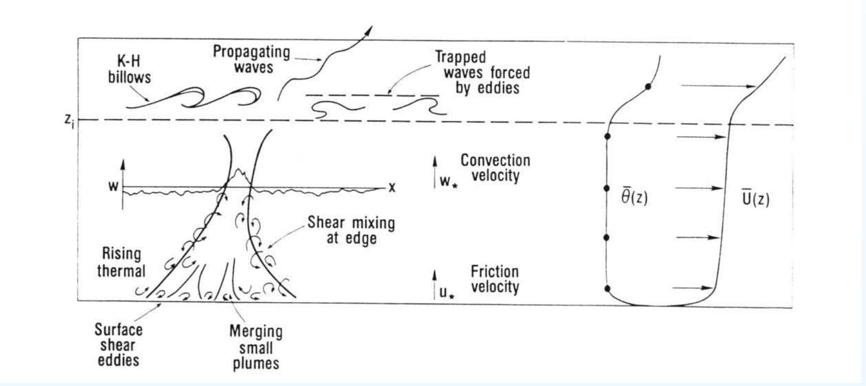
Convective Boundary Layer:
Ground-level cross-wind integrated concentration as a function of S, for $K = 3$.

Determination of a: conclusions

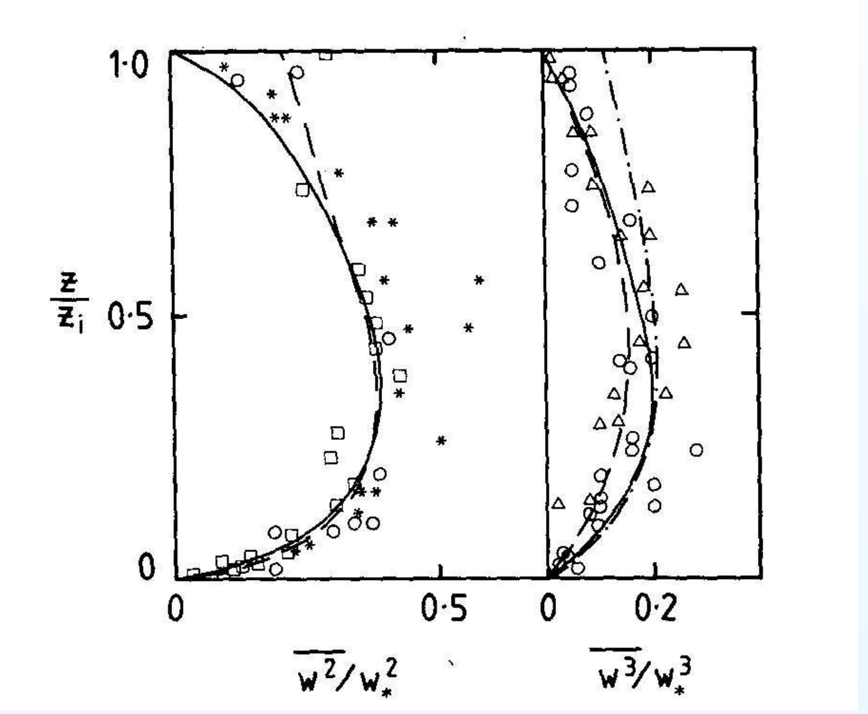
Summary of a determination:

- big unresolved issue of non-uniqueness
- non-Gaussianity of turbulence requires PDF “modelling”
- skewness-kurtosis parameter space is structured (suggests closure)
- dispersion properties depend on PDF for (given the same S and K)
- dispersion propertis depend on non-Gaussian features

LSM: an application to the CBL



(Hunt et al., 1988)



(Sawford and Guest, 1987)

LSM: counter-gradient transport in CBL

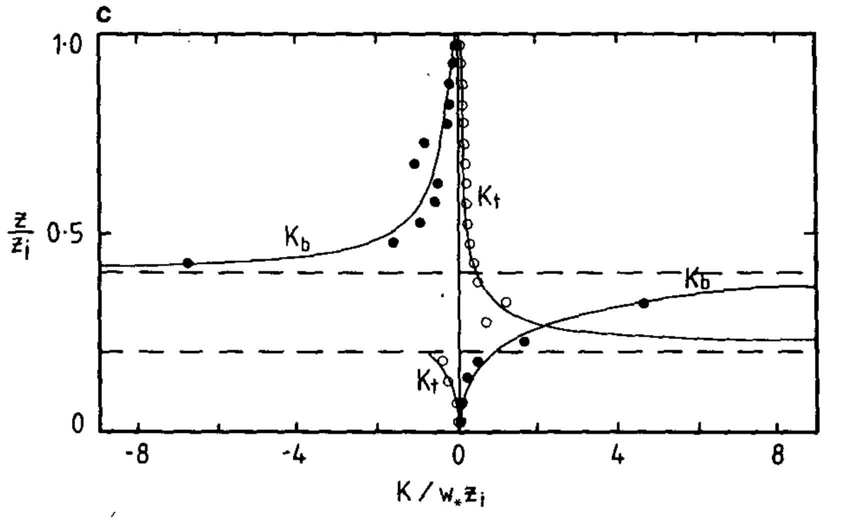


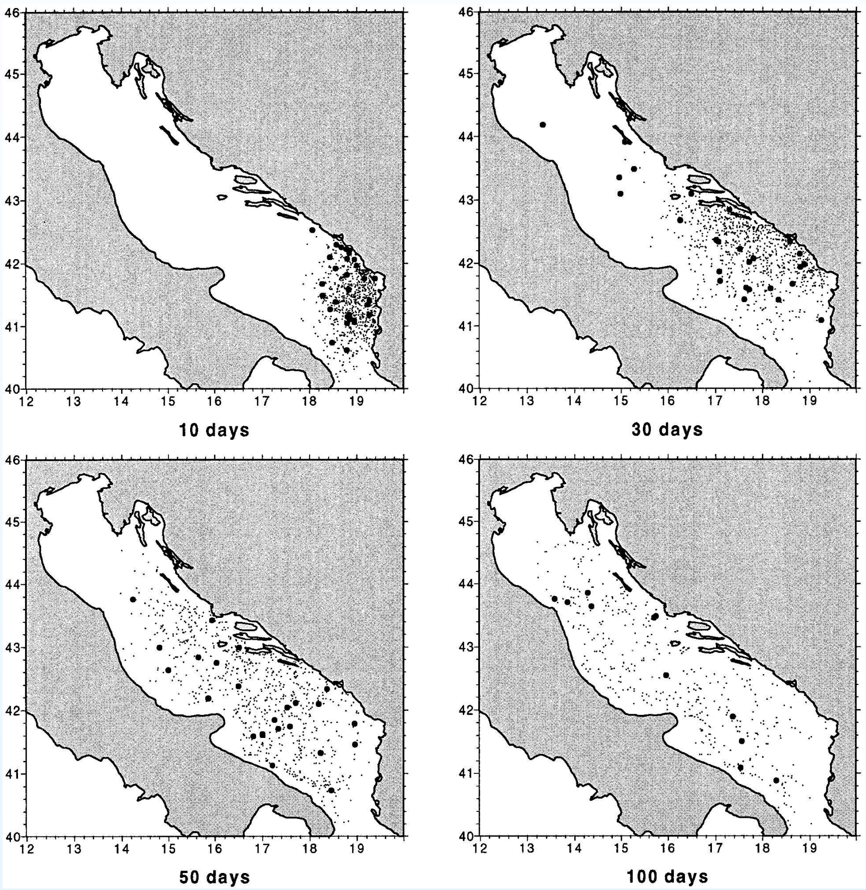
FIG. 7. (a) Flux, (b) gradient and (c) diffusivity profiles for continuous area sources at the top (subscript *t*) and bottom (subscript *b*) of the CBL. In (b) the broken lines show the LES results of Moeng and Wyngaard.

Effective diffusion coefficient

$$K_{\text{eff}} = -\frac{\langle w'c' \rangle}{\partial_z \langle c \rangle}$$

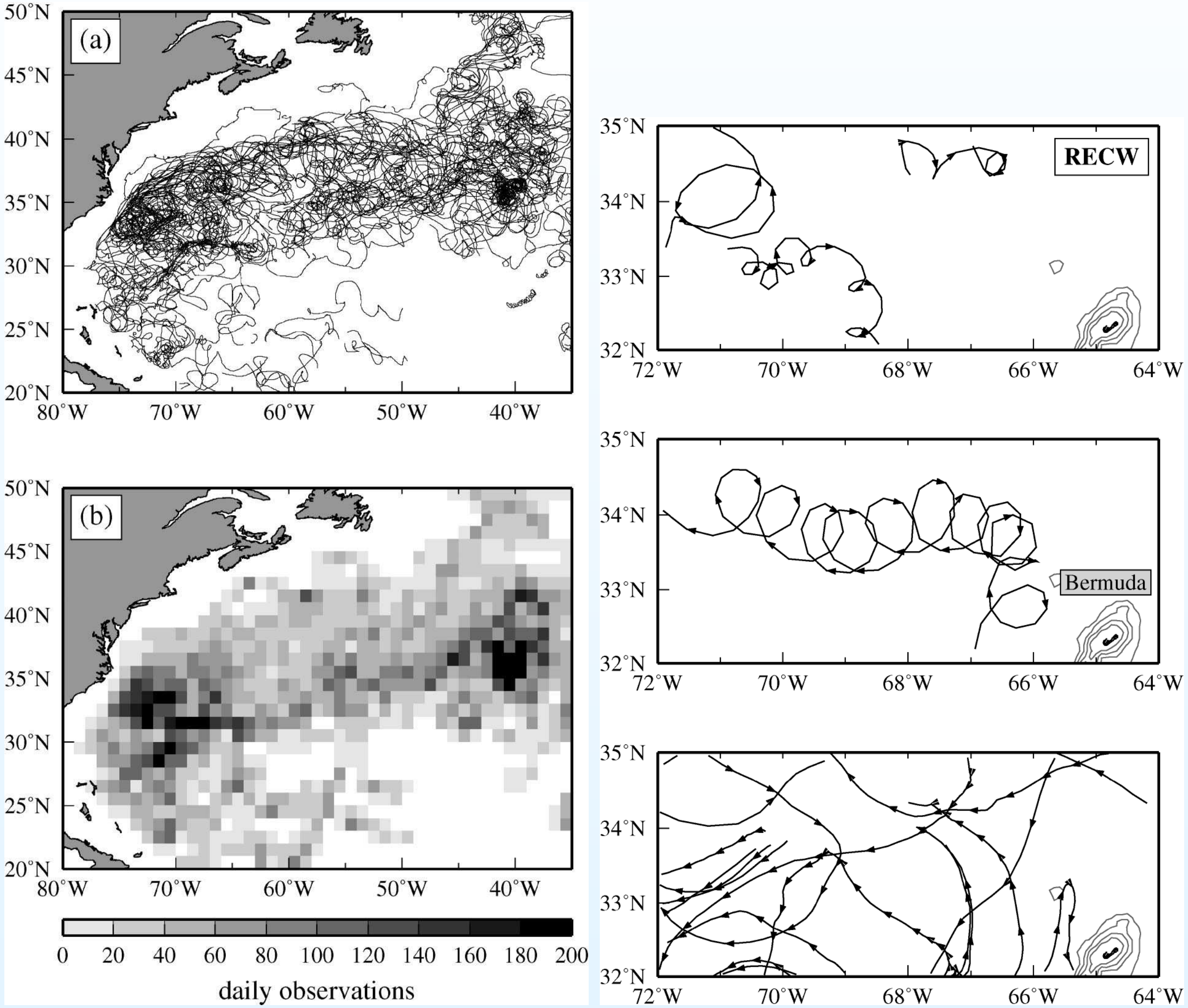
(Sawford and Guest, 1987)

LSM: an application to drifters in the Adriatic Sea



a Lagrangian transport model with parameters derived from the data has been implemented. Simulated particles have been compared with drifter data with positive results. The model is found to be able to reproduce reality with good approximation, except for a specific advective event during the late summer season. Finally, the residence timescale T, that is, the average time spent by a surface particle in the basin, has been estimated. Direct estimates from the data suggest T 70–90 days, but these values are biased due to the finite lifetime of the drifters. Model results have been used to estimate the bias, and they suggest a “true” value of T 200 days. (**?**)

Non-uniquenes and rotation: surface circulation in the ocean



Oceanic drifter trajectories (Veneziani et al., 2004).

Non-uniquenes and rotation: a simple model

The simplest form of the LS model arises when the spin term is taken to be linear in velocity

$$a'_i = \epsilon_{ijk}\Omega_i u_k$$

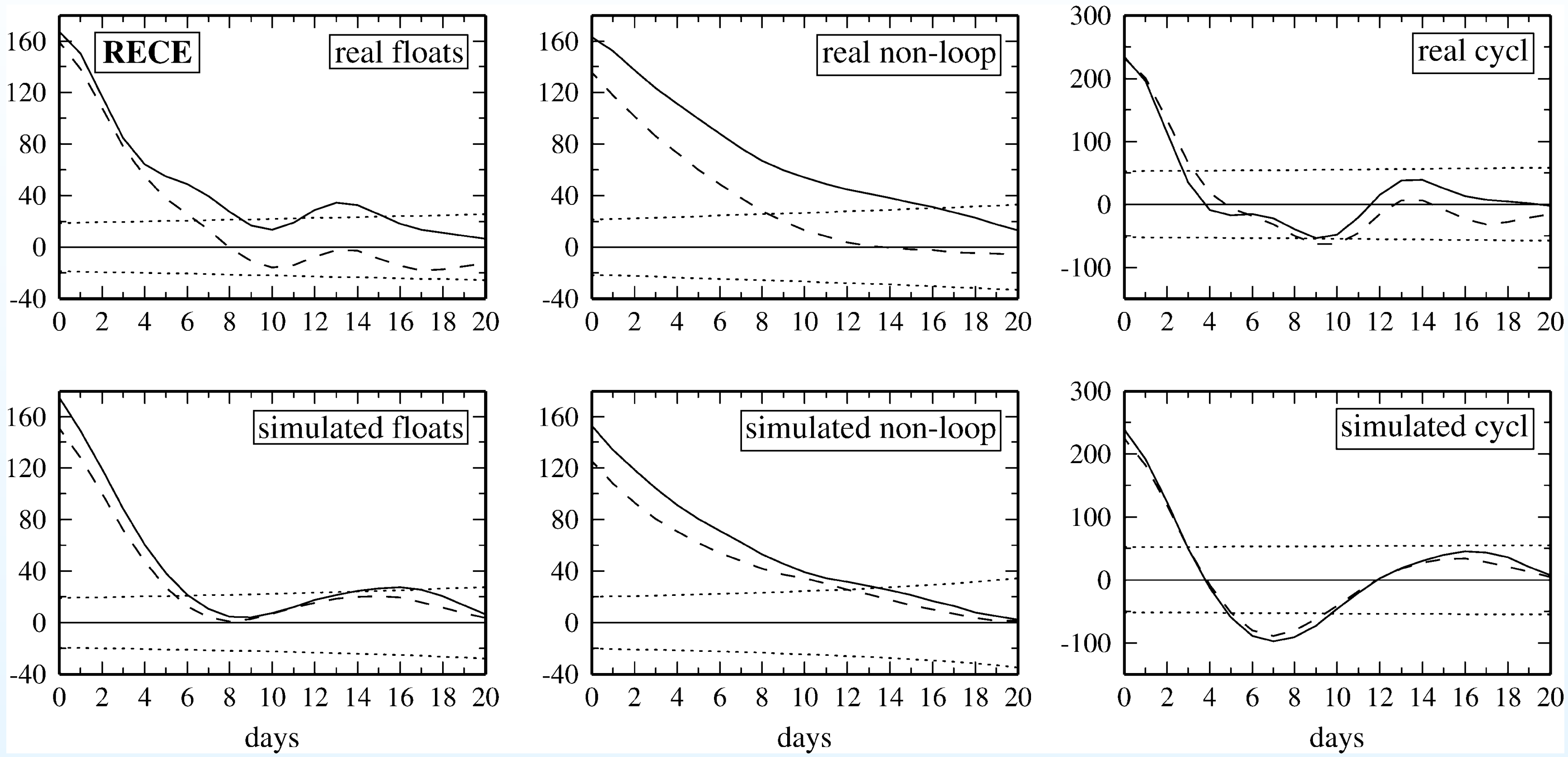
so that

$$\mathrm{d}u_i = \left[-u_i T_L^{-1} + \epsilon_{ijk}\Omega_i u_k\right]\mathrm{d}t + b_{ij}\,\mathrm{d}W_j$$

The autocorrelation functions are

$$R_{ii} = \exp\left(-t/T_L\right)\cos\left(\Omega t\right)$$

Non-uniquenes and rotation: results



Measured and simulated autocovariances (Veneziani et al., 2004).

Relative dispersion and two-particle LSM

Relative dispersion is a process that depends on the combination of the Eulerian and Lagrangian properties of turbulence.

Scales can be defined from Eulerian and Lagrangian structre functions

$$\langle v_i^2 \rangle = C_K(\varepsilon \Delta x)^{2/3} = 2\sigma^2 (\frac{\Delta x}{\lambda})^{2/3}$$

$$\langle u_i^2 \rangle = C_0(\varepsilon \Delta t) = 2\sigma^2 (\frac{\Delta t}{t})$$

(Maurizi et al., 2004b, 2006).

Relative dispersion: non-dimensional FPE

With σ, λ and τ, the FPE can be made non-dimensional

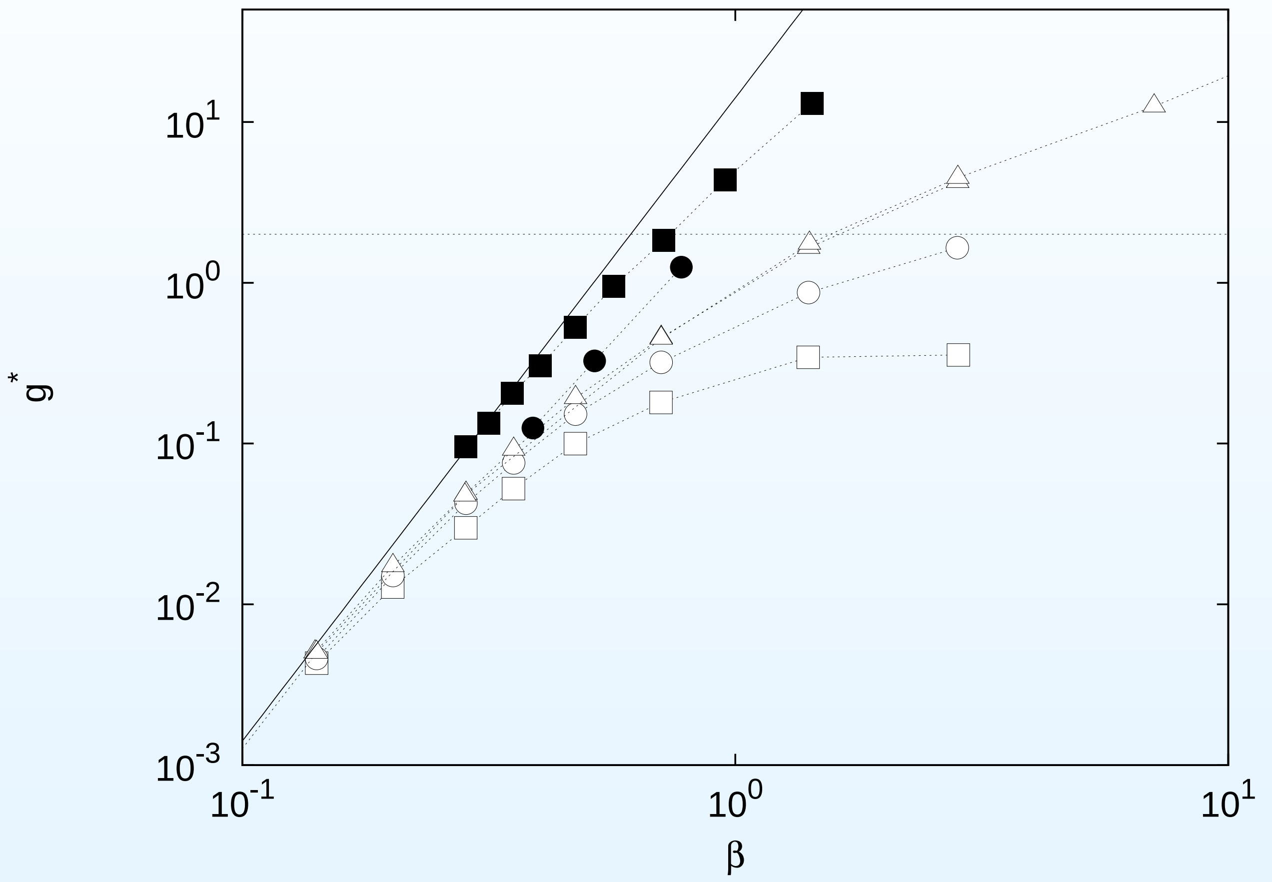
$$\partial_t p + \beta u_i \partial_{x_i} p + \partial_{u_i}[a_i p] = \partial_{u_i}\partial_{u_i} p +$$

where

$$\beta = \frac{\sigma\tau}{\lambda} = \frac{C_K^{3/2}}{\sqrt{2}C_0}$$

can be recognised as a version of the Lagrangian-to-Eulerian scale ratio.
Any model (once non-uniqueness is resolved) depends on β only.

Relative dispersion: non-dimensional results



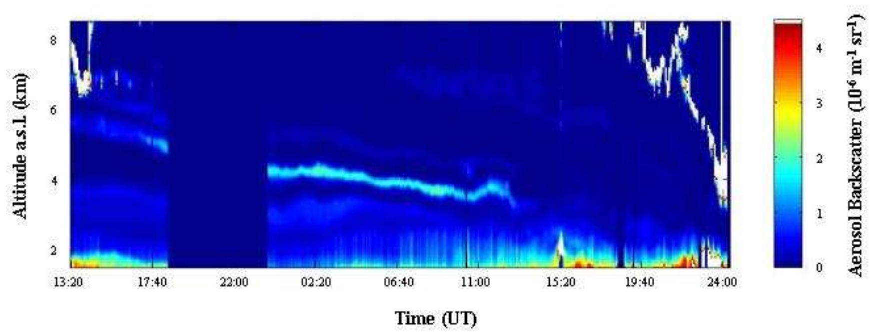
Variations of the normalised richardson coefficient $g^* = \frac{2g}{C_0}$ as a function of β for different models (Maurizi et al., 2004b, 2006).

Summary on Lagrangian dispersion modellig

- dispersion cha be modelled in the Lagrangian frame by using the Langevin equation (or, more generally, a Stochastic Differential Equation)
- The equivalence between Fokker-Plank and Stochastic Differential equations allows for the definition of a necessary condition to constrain an LSM to be consistent with Eulerian PDF of the flow
- knowledge on the PDF is limited (few moments) but non-Gaussianity is important
- non uniqueness of WMC models allows for the introduction of further dynamical constraints (rotation)
- two-particle (reltive dispersion) models can be also built and show interesting properties

Just to conclude: the Etna 2002 eruption case study

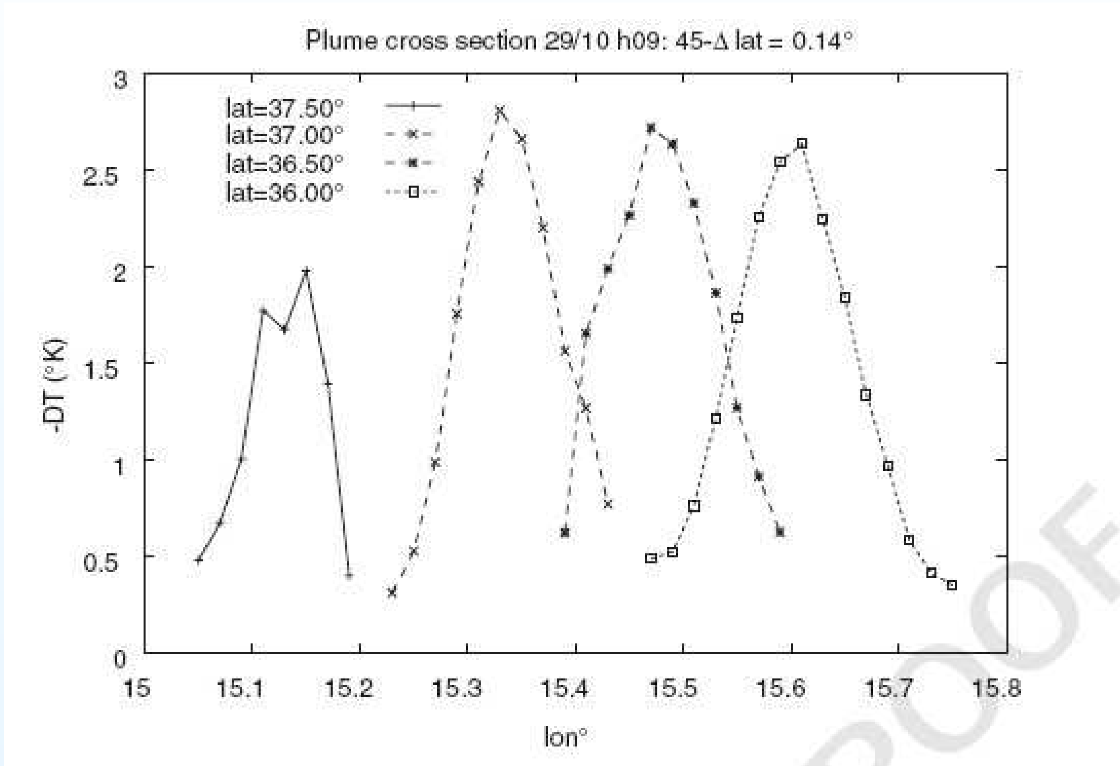
A case study: the Etna 2002 erup



In autumn 2002, Mount Etna, the largest European volcano, located in Sicily, Italy (37^o 440 N, 15^o E, 3350 m asl), awoke from a quiescence period, giving rise to a long series of strong seismic and eruptive events. From the first eruption on 27 October 2002 until January 2003, [. . .]

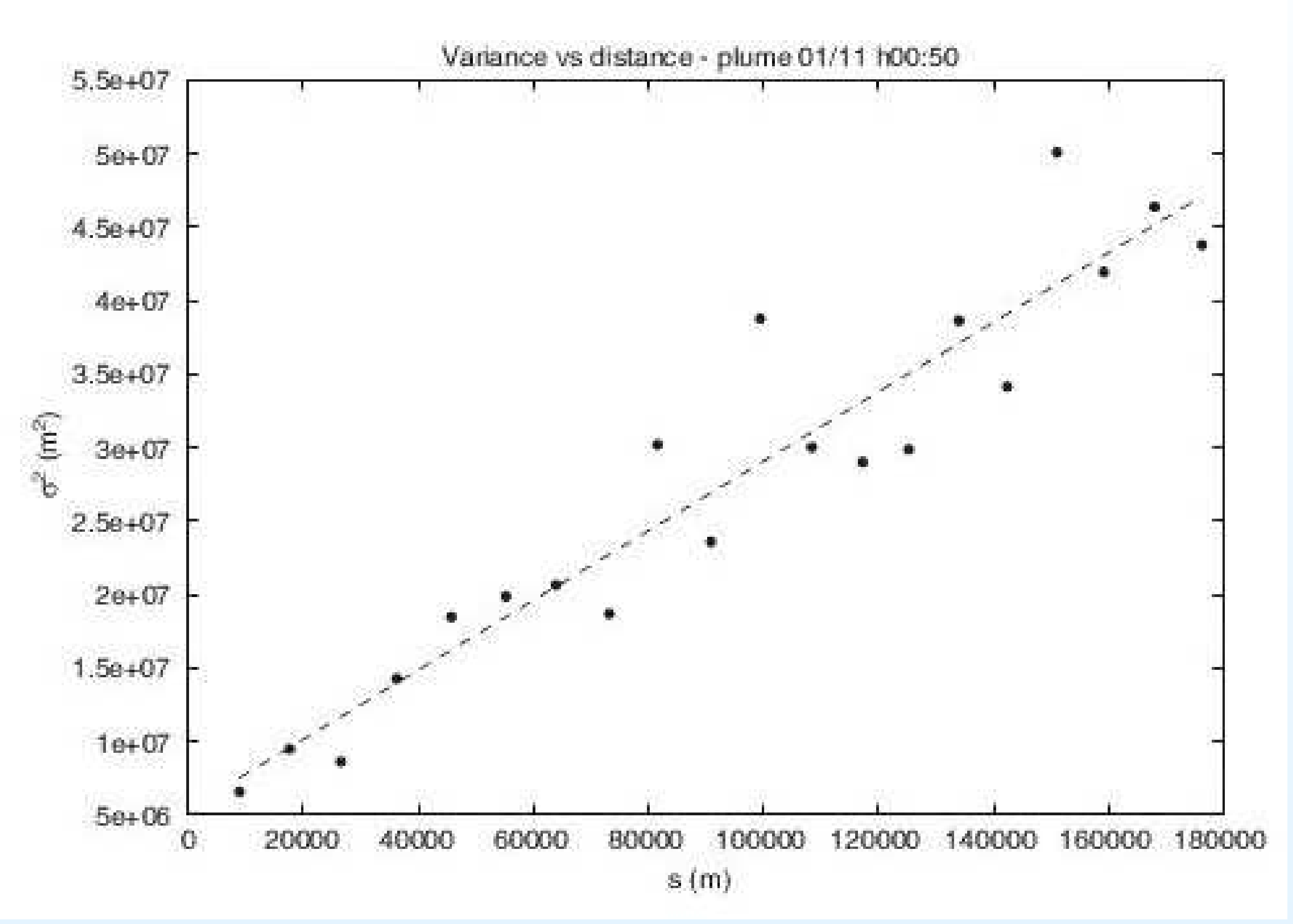
The lidar observations performed in Potenza, Italy, reveal the presence of aerosol layers made up of young sulfate particles and a low soot content, characteristic of the volcano's emission. Downward large-scale motion was measured, with a velocity larger than that due to gravitational sedimentation. (from Villani et al., 2006)

Etna 2002



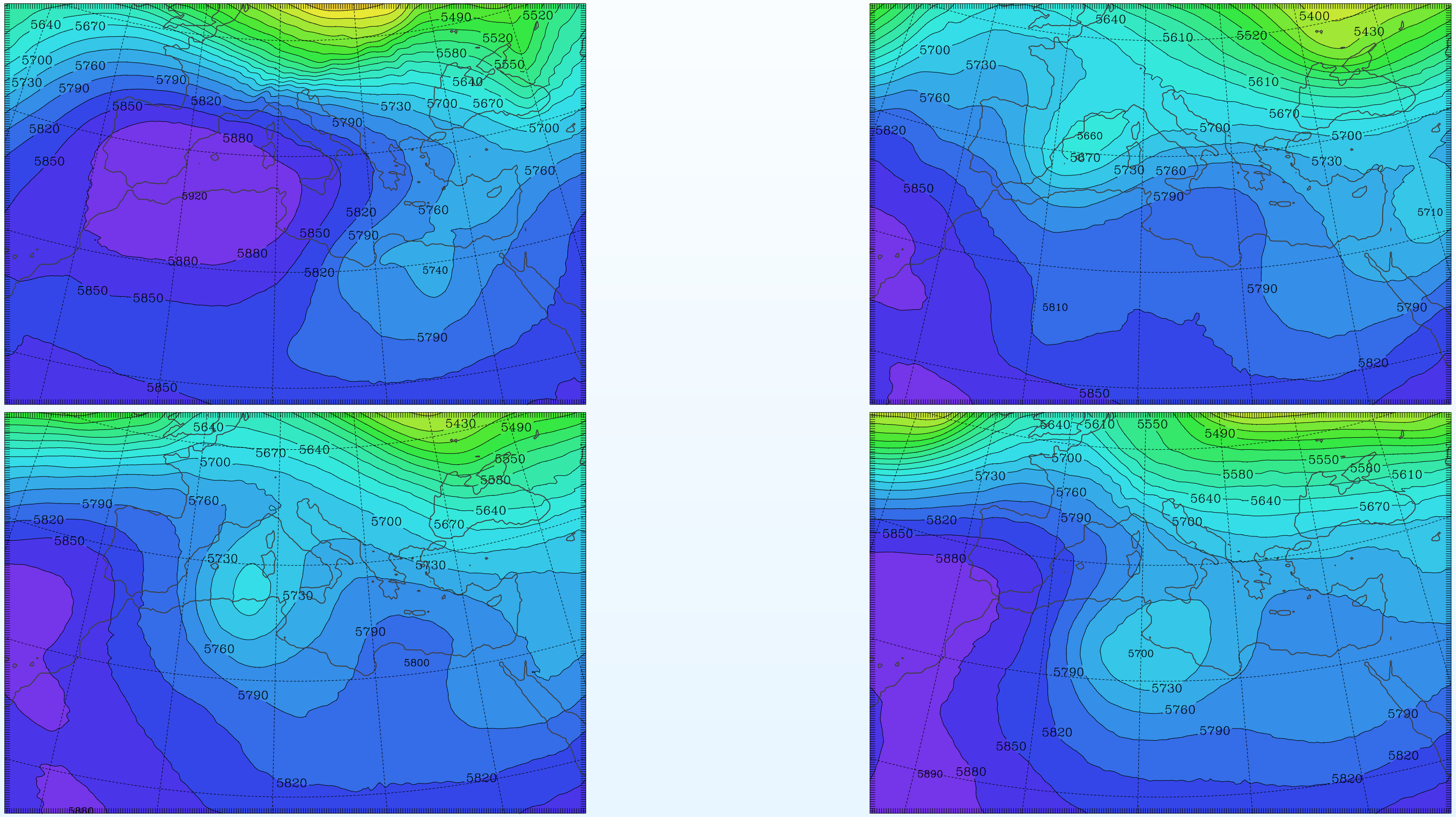
Cross section of the thermal image of the plume on 2002-10-29H09:45 (Tiesi et al., 2006)

Etna 2002



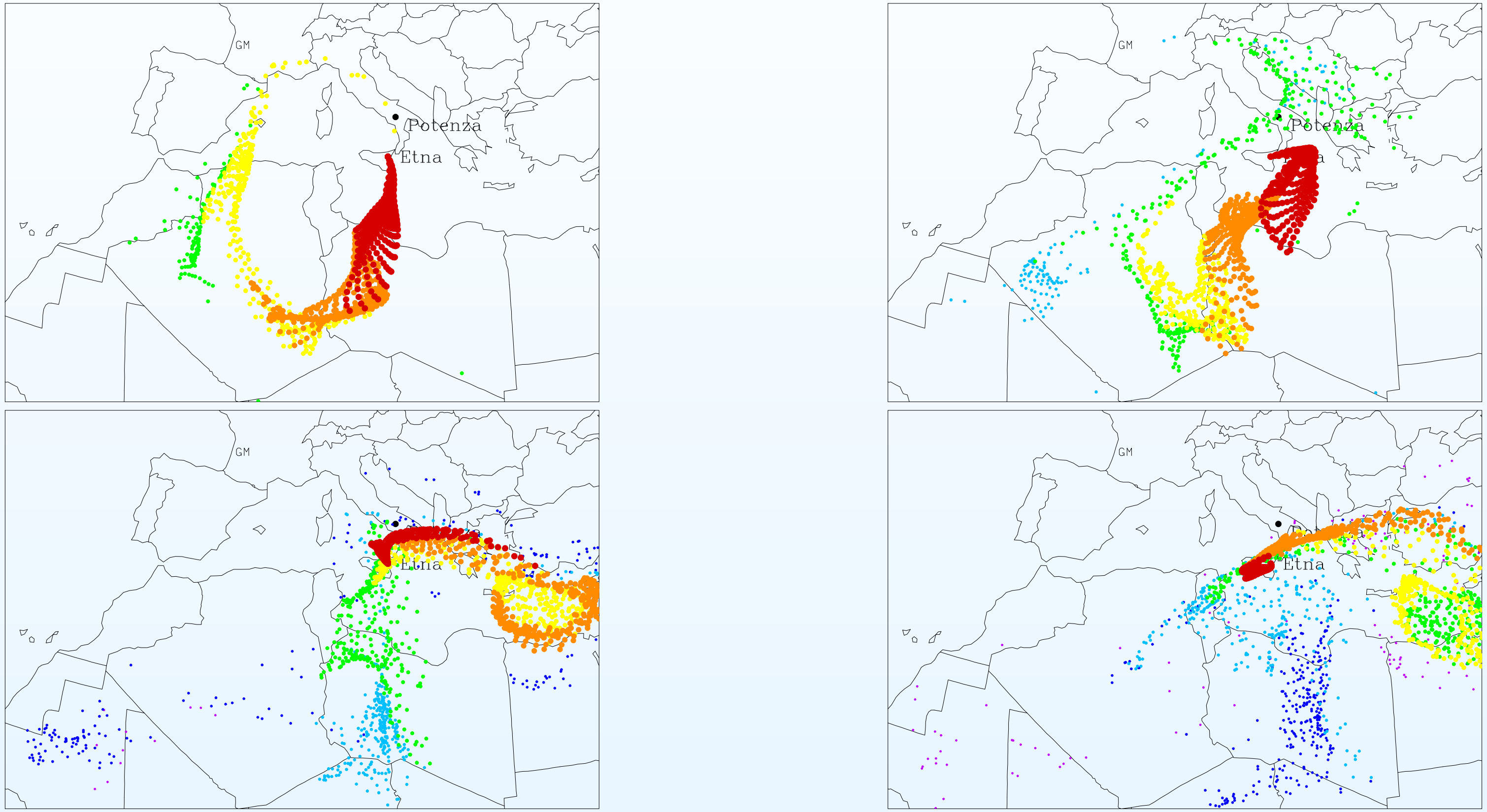
Increase of the horizontal spread (with respect to the center of mass)

Meteorological situation



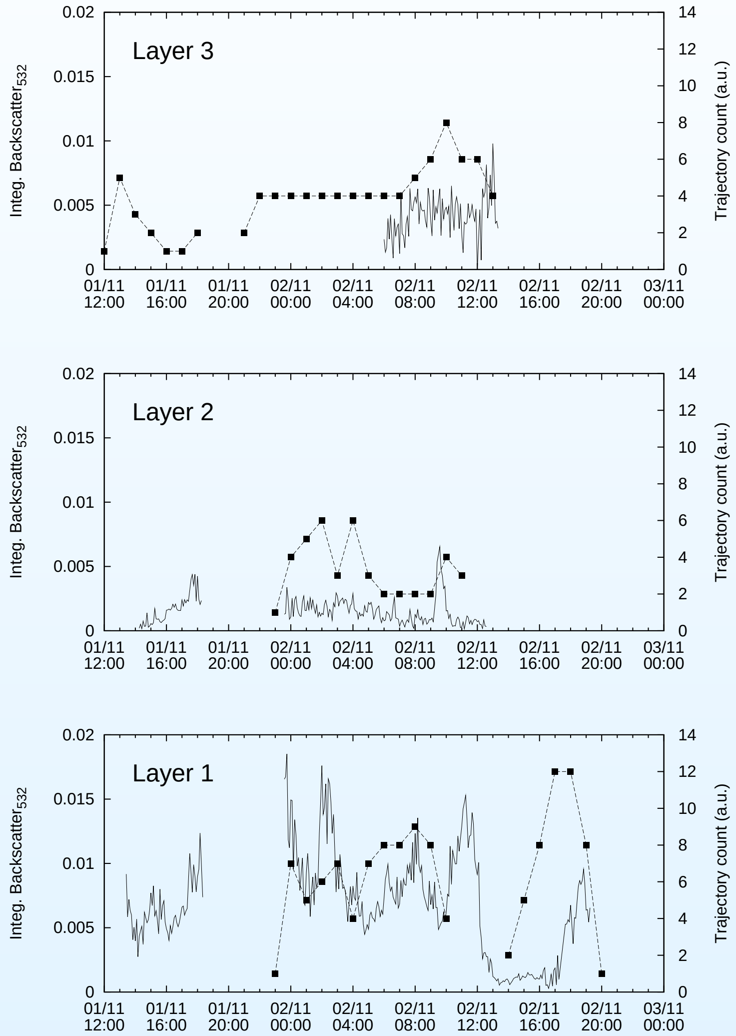
Meteorological maps of the geopotential height at 500 hPa. Top, left to right: 28 October 0000 UT, 31 October 1200 UT, 1 November 1200 UT, 2 November 1200 UT.

Forward particle trajectories



Selection of frames for the Etna plume evolution. From top, left to right: 30 October 0800 UT, 31 October 0900 UT; 1 November 1600 UT; 2 November 0000 UT; 2 November 2300 UT. Different colors denote the age of the particles: red) $<$ 1 day; orange) $<$ 2 days; yellow) $<$ 3 days; green) $<$ 4 days; cyan) $<$ 5 days; blue) $<$ 6 days; magenta) $<$ 7 days. (Villani et al., 2006)

Comparison with lidar measurements



Comparison of measured IB_{532} (continuous line) vs. number of computed trajectories (dashed line with black squares) over Potenza for the period 1 November 2002 (1200 UT) – 3 November 2002 (0000 UT). From top to bottom: Layer3, Layer2, Layer1. The scale for trajectory count is arbitrary and is the same for all the panels.

Updated version of this presentation

`http://ginevra.bo.isac.cnr.it/turbdiff/`

following the link lecture notes

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